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THE SYNCHROCYCLOTRON AT AMSTERDAM

I. GENERAL DESCRIPTION OF THE INSTALLATION

II. THE OSCILLATOR AND THE MODULATOR

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Reports on cyclotrons often have rather the appearance of aiming at overwhelming the public with amazement, with all their talk about metres thick magnet poles, hundreds of tons of iron, tens of kilowatts, velocities approaching the velocity of light, etc. Here, in this description of the Philips synchrocyclotron, it will be shown — without any attempt at belittling the impressiveness of the installation — how it comes about that in the building of a cyclotron one is in many respects obliged to adopt exceptional methods of construction.

I. GENERAL DESCRIPTION OF THE INSTALLATION

Philips have built a synchrocyclotron which has been installed in the Institute for Nuclear Research at Amsterdam, where it was officially taken into use on 10th November 1949. This apparatus is now working day after day and only a few interruptions have been necessary for maintenance. It is operated by Philips' personnel and for two-thirds of the time is at the disposal of Philips for their own research work and for the production of artificial radioactive substances used in all sorts of medical, biological and technical investigations. For the rest of the working time the beam of accelerated particles produced by this apparatus is available for scientific research in general. With the approval of the Institute also research workers at universities in the Netherlands can avail themselves of the opportunities offered by the synchrocyclotron for carrying out their investigations.

In this first article (I) a concise general description of the construction of this synchrocyclotron will be given, together with a brief account of what has so far been achieved with it. Further articles will deal with the most important component parts of the installation, the first of these (II) following immediately upon the general survey.

The theoretical principles of the cyclotron were discussed in an article (further referred to as A) which appeared in this journal some months ago [1]. For the benefit of the reader the most fundamental facts and formulae will be repeated here where necessary.

We shall start by giving in *figs 1a* and *1b* a schematic drawing of the chief part of the installation, the actual accelerating apparatus. *Fig. 2*, a photograph of a model of the apparatus in a partly dismantled state, will make it easier to understand the drawing.

The drawing shows the evacuated accelerating chamber *1*, in which the particles, in our case either deuterons (ions of heavy hydrogen) or alpha particles (doubly ionized helium atoms), emitted by the

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[1] W. de Groot, Cyclotron and synchrocyclotron, Philips Techn. Rev. 12, 65-72, 1950 (No. 3).

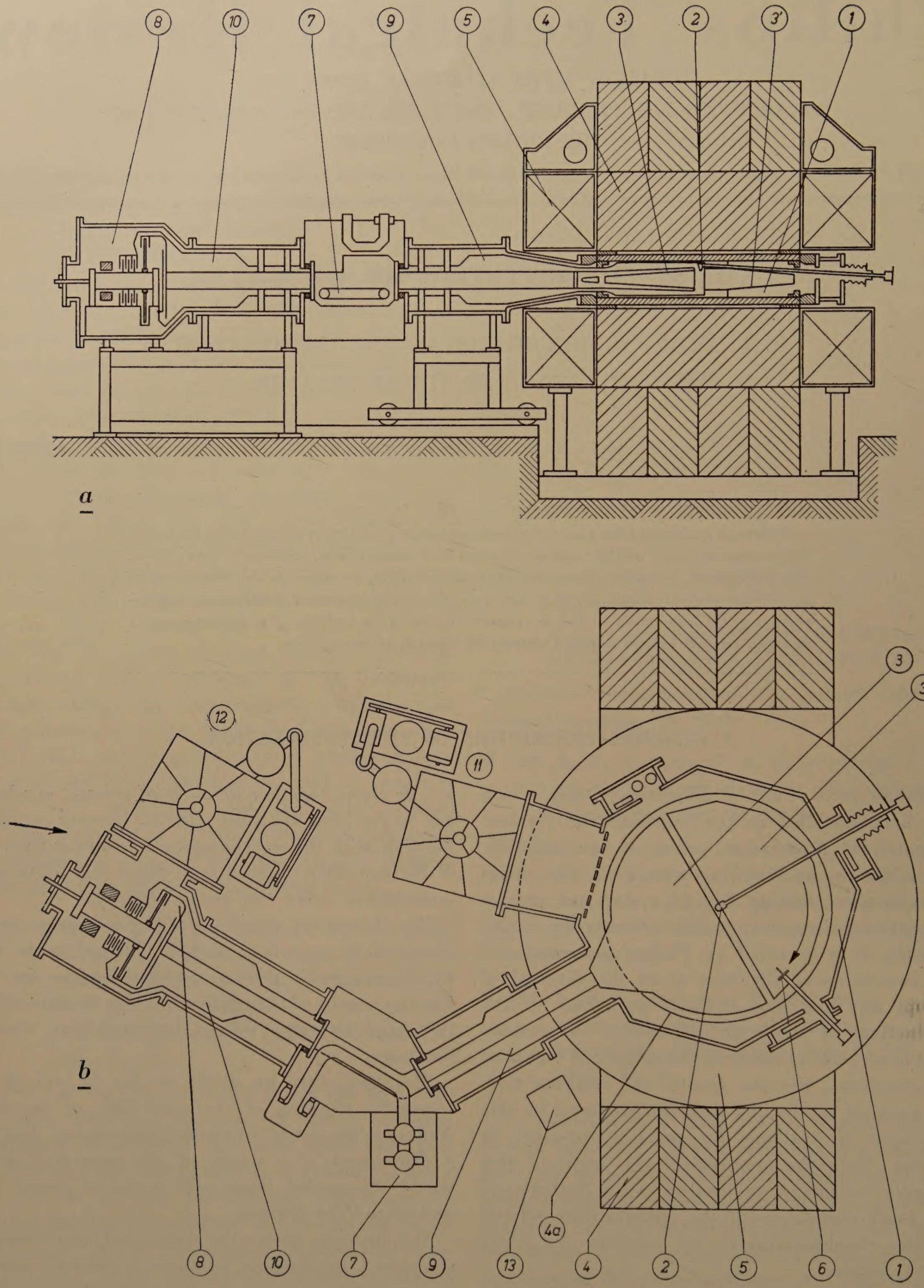

Fig. 1. Greatly simplified vertical and horizontal cross sections of the Philips synchrocyclotron at Amsterdam. (The vertical cross section, *a*, is really a juxtaposition of two cross sections along the centre lines of *9* and *10* in *b*.) *1* evacuated acceleration chamber, *2* ion source, *3* and *3'* dees, *4* magnet, *4a* pole face, *5* energizing coils, *6* target, *7* valves and other parts of the oscillator, *8* modulator, *9* and *10* coaxial transmission line, *11* and *12* vacuum pumps, *13* boron counter chamber.

ion source *2* are to be accelerated, by means of an alternating voltage of the order of 15 kV_{peak} between the dees (*3* and *3'*). The magnetic field of the large magnet *4*, energized with the aid of the coils *5*, causes the particles to describe a circular path, at such a rotational speed that they return each time in the right phase to the gap between the dees to undergo again an accelerating action from the alternating voltage, but with the radius of the orbit increasing with the increasing energy of the particles. The ultimate energy to be reached is determined entirely by the flux density B of the magnetic field and the ultimate radius r of the orbit [2]). In our case $B = 1.38$ Wb/m^2 (13,800 gauss) and $r = 78$ cm, so that deuterons can be accelerated to a final energy of 28 million electron volts and alpha particles to 56 MeV.

To generate the powerful magnetic field mentioned above in the large gap between the pole pieces (diameter of the pole pieces 1.80 m, distance between pole pieces 36 cm) 200 tons of iron was required for the magnet, whilst the energizing coils contain 32 tons of copper and are fed with 180 amperes. The electric power required to compensate the I^2R-losses is 80 kW and oil cooling is provided to dissipate this power. The great distance between the pole pieces (great also in comparison with similar installations) was chosen so as to be able to produce a very intensive beam of particles. This was particularly important for the object aimed at in the designing of this synchrocyclotron. The relation between pole distance and beam intensity is made clear when bearing in mind that, although by a suitable field variation the ideal path of the particles is stabilized in the centre plane of the gap, the particles may still oscillate vertically (and radially) about that ideal path. The more space is left for the vertical oscillations, i.e. the higher the accelerating chamber, the more particles can escape collision with the walls of the dees and thus participate in the acceleration right to the end.

The beam of accelerated particles is made to strike the target *6*, which can be inserted more or less deep into the accelerating chamber

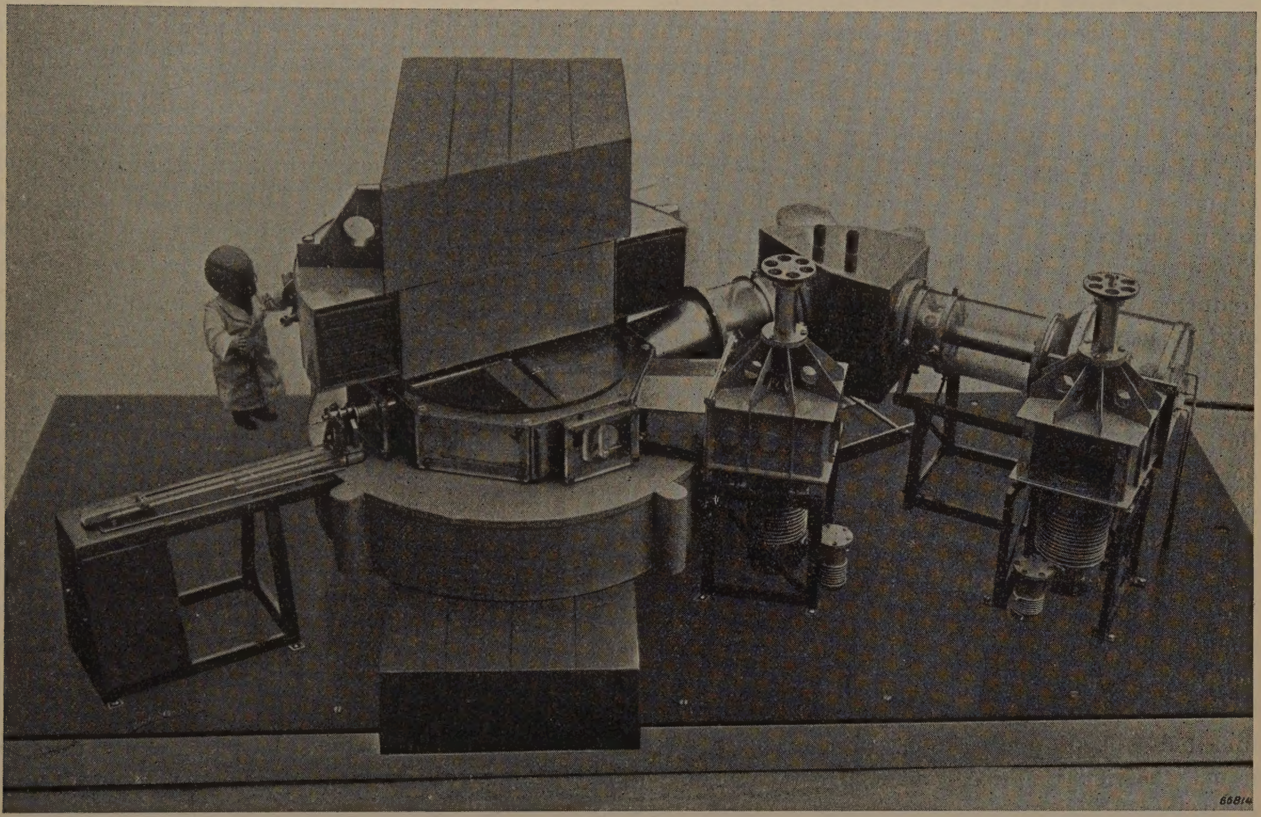

Fig. 2. View of the synchrocyclotron, with the magnet partly cut away to show the acceleration chamber with the two dees.

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[2]) See, e.g., Philips Techn. Rev. 11, 70, 1949 (No. 3).

to adjust the energy of the particles collected. The materials to be transmuted can be put on the target. Due in part also to a suitable design of the ion source, it has already been possible to increase the beam intensity of our synchrocyclotron so that the average intensity of current carried by the charged particles (deuterons) to the target amounts to 20 micro-amperes. This is to be regarded as a high value. With this current on the target an energy of $28\times 10^6 \cdot 20\times 10^{-6} = 560$ watts is converted into heat, which has to be dissipated by water cooling (it is by measuring this heat that we get to know the current strength). Also the dees and many other parts are cooled with water.

The alternating voltage required between the dees is obtained with the aid of an oscillator (*7* in fig. 1). For the acceleration of particles with mass m and a charge q the alternating voltage must have an angular frequency of $\omega_E = qB/m$ (see article A). For deuterons as well as for alpha particles the frequency required, with the aforementioned value of B, is therefore 10.7 Mc/s. For accelerating protons twice this frequency would be needed.

Since it seemed impracticable at the time to make the oscillator suitable for both these frequencies, one of the first steps in the designing of our synchrocyclotron was to decide with what particles it was to be worked. Fortunately this choice is not very critical: both with deuterons and with protons a large number of nuclear reactions can be brought about which yield a great variety of radioactive substances, among which are materials that cannot be produced from a uranium pile. Weighing the usefulness of protons against that of deuterons, it may be said that with an energy of some tens of MeV deuterons are certainly more suitable for producing neutrons and that in general they may perhaps offer more universal possibilities of application. Anyhow there was no doubt that if the (lower) deuteron frequency were chosen there would be fewer difficulties to overcome in the construction of the oscillator and, particularly, of the modulator, so that in our case it was decided in favour of the deuterons.

The modulator is indicated in fig. 1 by *8*. Let us recall briefly the function of this important part (a detailed explanation is given in A). As soon as the particles reach a high energy in the accelerating chamber they have a tendency to "get out of step": firstly because they get closer to the edge of the pole pieces, where the flux density B must necessarily be smaller than in the middle in order to stabilize the path of the particles, and secondly because the relativistic increase of their mass then becomes noticeable. Owing to these two causes the condition for synchronism $\omega_E = qB/m$ is then no longer fulfilled, the angular frequency ω_E being too high for the particles at the edge. This limits the final energy attainable with the classical cyclotron. When, however, ω_E is gradually reduced — the principle of the synchrocyclotron — the particles can be accelerated to greater final energies. The desired frequency variation is brought about with the modulator. This apparatus contains a capacitor the value of which varies periodically with time and acts on the tuning of the oscillator. In that part of each modulating cycle in which the oscillator frequency decreases, a stream of particles is accelerated. An important feature is the phase stabilization that takes place, in consequence of which it is not necessary that the reduced frequency should at every instant be exactly in step with the changing situation as the radius of the orbit of the individual particles increases. It is due to this that with the synchrocyclotron powerful intermittent beams of accelerated particles can be obtained.

The modulation frequency in our apparatus is 2000 c/s, which means to say that 2000 times per second a group of accelerated particles reaches the target. This relatively high modulation frequency, which is favourable for a high average intensity of the beam current — the number of accelerated particles per group being in the first instance independent of the modulating frequency —, involved no great technical difficulties on account of the fact that in our case the frequency sweep need only be 4%. In fact, from the middle to the edge the flux density drops in our apparatus by about 2.5%, and the mass increase of a deuteron having received a kinetic energy of about 30 MeV at the edge is about 1.6% (the rest energy of the deuteron is 1840 MeV), so that with a frequency drop of about 4% the condition for synchronism $\omega_E = qB/m$ is again fulfilled at the edge [3]).

For the sake of comparison some figures are given relating to the synchrocyclotron at Harwell in England, which was put into service some time ago [4]). This accelerates protons to an energy of about 180 MeV. The relativistic mass increase is then already about 20% and the frequency sweep has to be about 30%. The modulation frequency has for the time being not been made greater than 80 c/s.

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[3]) This is likewise the case for alpha particles, since the rest energy and the maximum kinetic energy obtained are in this case both twice these energies of the deuteron, so that the mass increase is again about 1.6%.

[4]) T. G. Pickavance, J. B. Adams and M. Snowdon, The Harwell cyclotron, Nature **165**, 90-91, 21st January 1950.

It is well to emphasize once more (cf. A) that the higher final energy attainable is not the only advantage of the synchrocyclotron compared with the classical cyclotron. In our case the final energy of the deuterons is 28 MeV, whilst with a classical cyclotron deuterons can also be accelerated up to 25 MeV with a not unreasonably small efficiency. The synchrocyclotron, however, requires a much lower oscillator power for such a final energy and also in other respects the construction is much simpler than that of a classical cyclotron, as will be seen in the next article in this series.

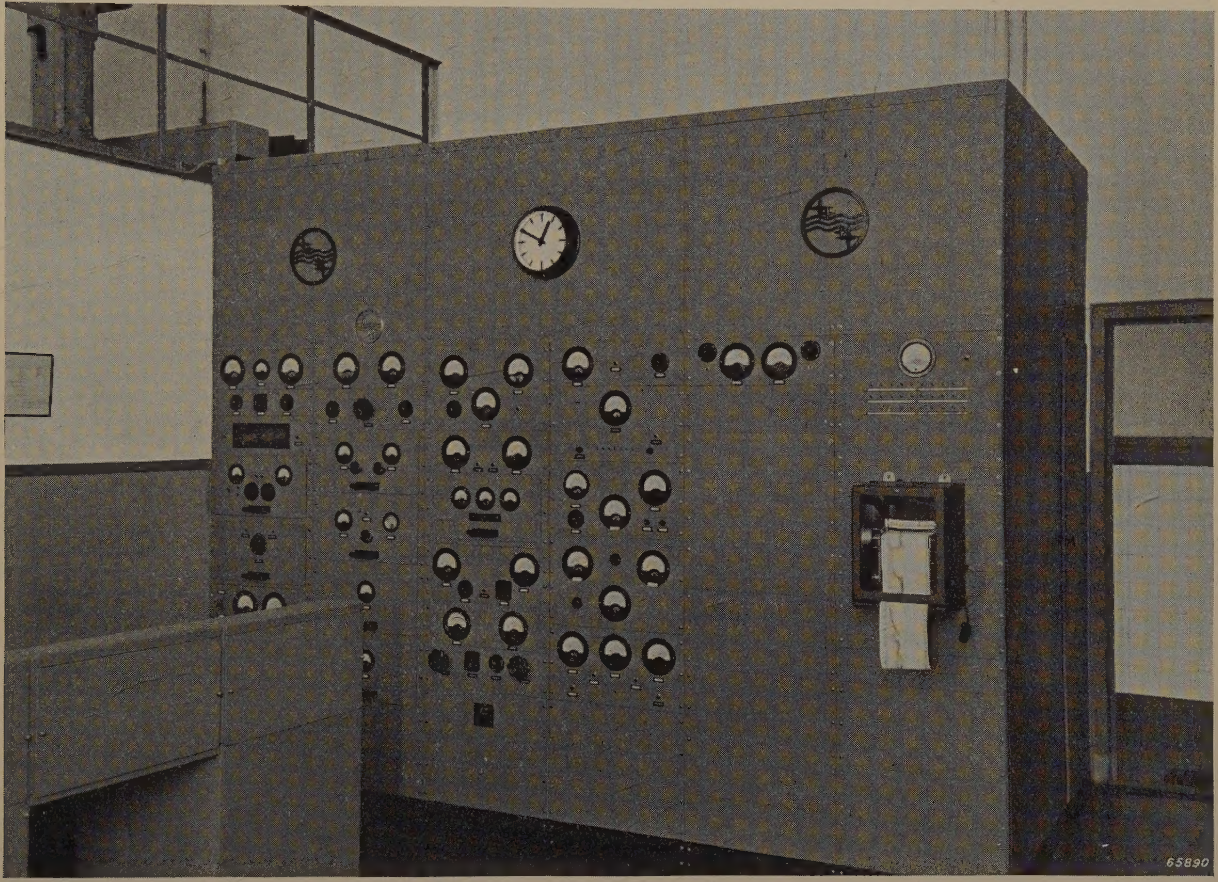

Fig. 3. Part of the control and operating room of the synchrocyclotron at Amsterdam. The personnel in this room are protected against the radiation from the accelerating apparatus by a layer of water 3.5 m thick.

Let us turn back to fig. 1 again. The high-frequency oscillation is applied to the dees and the modulator via a coaxial transmission line, *9* and *10*; the fact is that at the frequency used (wavelength about 30 m) and with the large dimensions of the parts — to begin with the accelerating chamber —, the system can no longer be regarded as a lumped circuit. The whole system is set up with an angle in the middle instead of in one straight line; the only reason for this is that, for the purpose of maintenance, part *9* of the transmission line with the dee *3* should be easily removable. To that end the part *9* is mounted on an undercarriage running on rails laid radially to the magnet. After the part *7* of the oscillator has been removed parts *9* and *3* can be run out without having to shift part *10* of the transmission line and the modulator.

Further it is to be seen in fig. 1 that there are two vacuum pumps with their backing pumps, one (*11*) connected to the accelerating chamber, the other (*12*) to the modulator. The reason why one pump alone was not sufficient will be seen later.

Finally the synchrocyclotron installation comprises all sorts of parts, some of them very bulky, corresponding to what is needed for a radio transmitter, these parts not being visible in figs 1 and 2. For instance, there is a rectifying installation supplying the direct voltage of 12 kV for the oscillator valves; a drum with 2 × 10 metres of rubber hose for taking up the potential difference of the cooling water, coming from the water mains, and the cooled anode at a high positive potential; a control and operating room, with a large number of controls and measuring instruments (*fig. 3*). This room is separated from the cyclotron by a water tank 3.5 m thick protecting the personnel against the neutron and gamma rays generated in the accelerating chamber (in the target and the walls). On the other sides the cyclotron is enclosed by concrete walls 1 m thick. When entering the cyclotron compartment, after the apparatus has been switched off, to collect the products of the transmutations by withdrawing the target from the accelerating chamber, the necessary precautions are of course also taken. The target is fixed to a holder 1.25 m long. Each holder, after having been used for an irradiation, is first

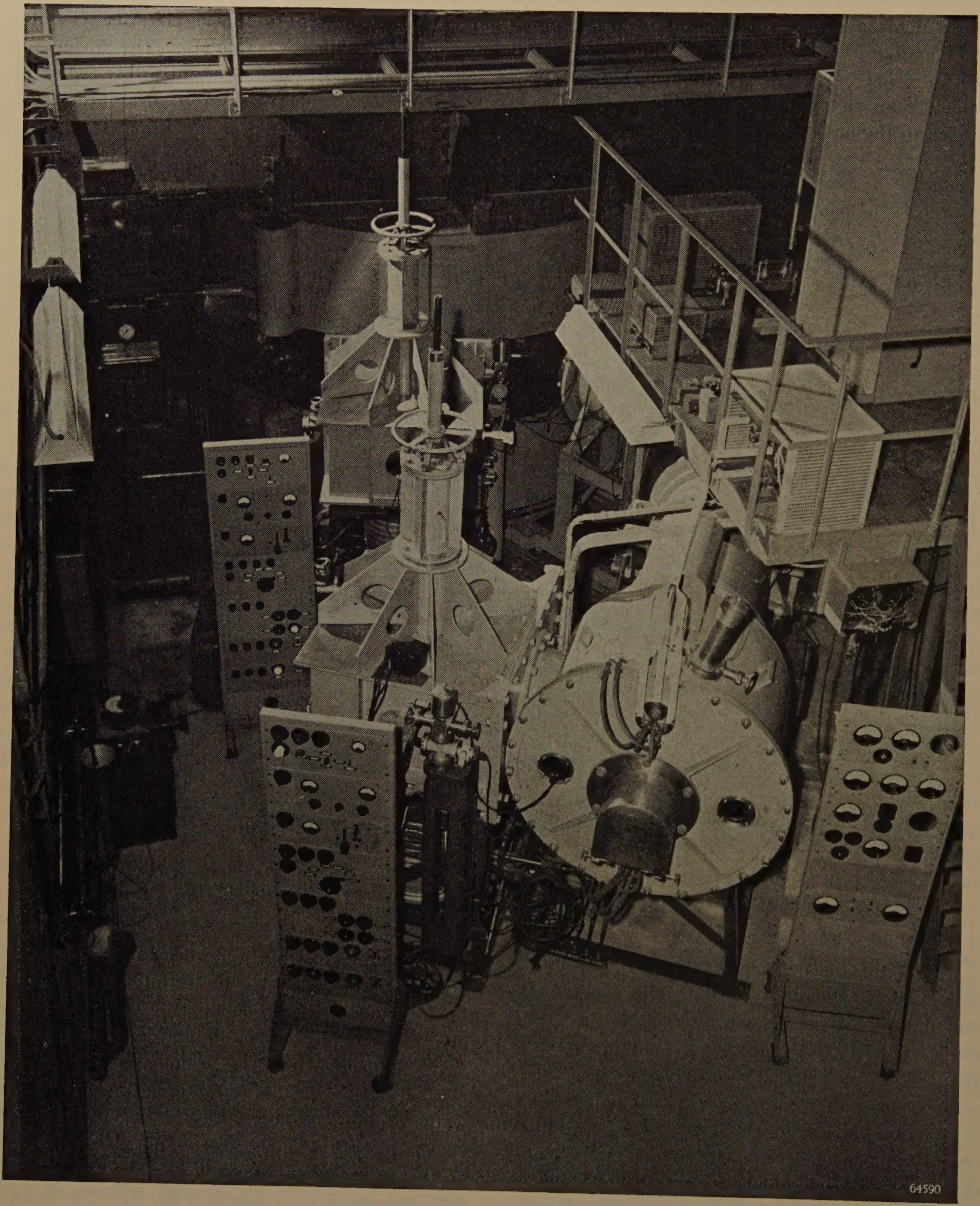

Fig. 4. The synchrocyclotron viewed in the direction of the arrow in fig. 1*b*. In the foreground on the right is the modulator.

left to "cool down" between blocks of concrete for a certain length of time (depending on the "life" and the concentration of the radioactive isotypes formed) before being used again.

The materials are usually irradiated until a radioactivity of some tens of millicuries [5]) has been generated in them. The neutron radiation emanating from the target (and other parts) is being continuously measured with a boron counter chamber [6]) mounted in a fixed position close to the accelerating chamber (*13* in fig. 1).

The photograph in *fig. 4* finally gives a view of the cyclotron as seen in the direction of the arrow in fig. 1*b*. This and the photograph in fig. 3 also give some idea of the extensive accessory apparatus that is needed. In the foreground on the right of fig. 4 is the modulator, which will be discussed in detail in the article now following.

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[5]) With phosphorus, for instance, an activity of 150 microcuries per μAh is obtained, thus in this case our beam of 20 μA yields 3 millicuries per hour. The unavoidable losses of radioactive phosphorus in the chemical processing required before the product can be supplied to the "customer" are already discounted in this figure for the yield. (The loss of activity during the chemical processing is negligible considering that the half-value time of this atom is about 14 days.)

[6]) This contains a solid or gaseous boron compound from which alpha particles are released by neutrons. The average ionization current produced by the alpha particles is measured.

II. THE OSCILLATOR AND THE MODULATOR

Principles underlying the construction of the oscillator

Owing to the phase stabilization of the particles travelling round in the synchrocyclotron these can be allowed to describe a large number of revolutions without risk of too many being lost. Consequently for a certain final energy a small gain per "loop" suffices. This is the reason why the alternating voltage between the dees in a synchrocyclotron of the dimensions chosen by us need not be greater than, say, 15 $\mathrm{kV_{peak}}$, whereas for a classical cyclotron of the same dimensions a voltage of certainly 100 $\mathrm{kV_{peak}}$ would be required between the dees.

Much smaller voltages than 15 $\mathrm{kV_{peak}}$ are also possible, but then there is the objection that the permissible modulating frequency and thus the strength of the beam current likewise become much smaller; this will be reverted to later.

This so much lower voltage has two very important advantages. In the first place there is much less risk of a breakdown of the insulators and disruptive discharges through the gas in the accelerating chamber, and this is an advantage that can be turned to good account in various ways. A higher gas pressure can be permitted in the accelerating chamber, thus making it possible to get greater ion concentrations and therefore more powerful beam currents. Further, one of the two dees can be earthed (thus the full dee voltage, instead of half of it, comes to lie between the other dee and the earthed chamber walls), thereby making it much easier to mount the ion source, the target, etc.

The second advantage of the lower dee voltage is that a much smaller high-frequency power is required than is the case with the classical cyclotron. If the dee circuit has a quality factor Q and a capacitance C (it is imagined for a moment as being replaced by a lumped L-C circuit), then a power

$$P = \frac{\omega C V^2}{2Q} \quad . \; . \; . \; . \; . \; . \; . \quad (1)$$

is needed to maintain, in the circuit, an oscillation with peak voltage V and angular frequency ω. (The index E used with ω in article A will henceforth be omitted.) The capacitance C is determined mainly by constructional requirements; in our case it could not be kept below 400 pF. Careful construction allowed of the quality factor Q being raised to 1500. With $\omega = 2\pi \times 10.7 \times 10^6$ and $V = 14$ kV we therefore get $P \approx 1.6$ kW for the dee circuit. The modulator circuit takes up an equal amount of power, so that the high-frequency power required for our synchrocyclotron totals somewhat over 3 kW, as compared with about 100 kW for a classical cyclotron of the same dimensions.

In *fig. 1* a very schematic representation is given of the oscillatory circuit. It may be regarded as a transmission line with two, large, lumped capacitances at the ends: the (non-earthed) dee in the accelerating chamber at one end, with the capacitance $C_1 = 400$ pF, and at the other end an equally large capacitance C_2 (the modulator M). With the aid of an oscillator the transmission line is caused to oscillate in such a way that a standing wave occurs with at each end a voltage anti-node and in the middle a voltage node (see the sketch). Thus the system forms a half-wave line. Owing to the influence of the lumped capacitances, however, the transmission line need not be 15 m long, which would correspond to the wavelength of 30 m, but only about 7 m. By periodically varying the capacitance C_2 of the modulator, the frequency is made to fluctuate periodically between about 10.7 and 10.3 Mc/s (variation of 4%, see I).

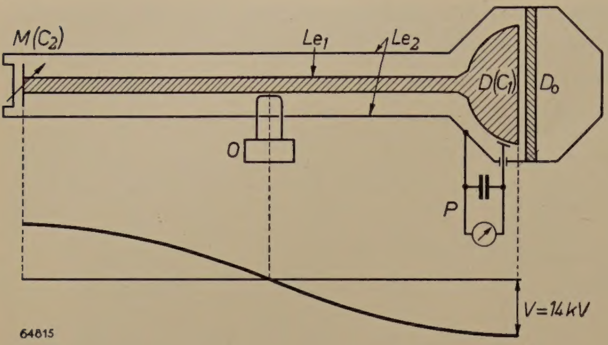

Fig. 1. Very schematic representation of the high-frequency oscillatory circuit of the synchrocyclotron. Le_1, Le_2 coaxial transmission line, *D* dee, *M* modulator, *O* oscillator valve with coupling loop. The other dee (D_0) being earthed, it can in principle be regarded as a narrow strip connected to the walls of the accelerating chamber and placed opposite the edge of the dee under high tension. Where in future we speak of "the dee" this is to be taken as meaning the non-earthed dee. *P* is a device for measuring the dee voltage. Below this drawing a curve has been plotted showing the voltage distribution along the system.

In order to minimize radiation losses the transmission line has been made in the form of two coaxial conductors, the inner one with a diameter of about 15 cm (of course this inner conductor itself is made in the form of a hollow pipe, since the high-frequency currents flow only along the surface), the other one with a (average) diameter of 60 cm. The peculiar configuration of the transmission line made it impossible to compute precisely in advance the dimensions required for the desired frequency of 10.7 Mc/s. The dimensions of the transmission line and the modulator were therefore determined experimentally with a model (the dimensions of the dee are governed by other considerations, viz. the need to avoid disruptive discharges and to provide as much space as possible for vertical deviations of the travelling particles), for which purpose a full-scale rough model was made which could easily be altered and with which the resonant frequency was determined after every modification. The final form of the model served as pattern for the actual construction, which of course differed from the model in its precise finishing: all parts are made of copper, perfectly fitting and soldered with silver, so that the dissipative resistance of the whole line is no more than 0.05 ohm. In this way the quality factor 1500 was reached.

The necessity of keeping the dissipative resistance so small is evident from a calculation of the current strengths occurring in the system: to charge the 400 pF capacitance C_1 (the non-earthed dee) to a peak voltage of $V = 14{,}000$ volts at a frequency $\omega = 2\pi \cdot 10^7$ a current is needed with the peak value

$$I = \omega C_1 V,$$

i.e. around 350 amperes. With the small total dissipative resistance mentioned, we thus arrive at the previously mentioned total dissipated power of somewhat more than 3 kW. Since the stream of particles impinging on the target generates about 560 watts (it may be possible to increase this figure still further by improving upon the ion source), we may say that about 15% of the high-frequency power is utilized, which for a cyclotron is a remarkably high "efficiency".

The circuit diagram of the oscillator is given in *fig. 2.* A Meissner circuit has been used, i.e. the anode circuit of the transmitting valve *B* is inductively coupled to the oscillatory circuit (drawn here as a lumped *L*-*C* circuit for the sake of simplicity), whilst also the reaction to the grid circuit (excitation) is inductive. The anode and grid coupling coils each consist of only one loop, placed beside the inner conductor of the transmission line in such a way that the magnetic lines of force of the loop envelop the conductor, and vice versa.

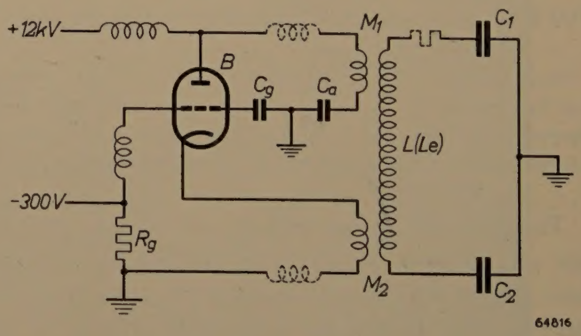

Fig. 2. Circuit of the oscillator. The oscillatory circuit of the synchrocyclotron can be regarded as an *L*-*C* circuit with the inductance mainly concentrated in the transmission line (*Le*) and the capacitance mainly in the dee (C_1) and in the modulator (C_2). This circuit is inductively coupled to the anode and grid circuits of the transmitting valve *B* (Meissner circuit). Undesired but unavoidable impedances are shown in broken lines.

The Meissner circuit had to be chosen because it is not possible to connect the inner conductor of the transmission line at the point of coupling to a part that has to have a defined (high-frequency) voltage. It is true that at that point, in the middle of the transmission line, there is a voltage node (cf fig. 1), but, owing to the periodical variation of the capacitance C_2 of the modulator, the position of the node is not fixed but shifted a little to and fro along the line. With the inductive couplings according to fig. 2 no trouble is experienced from this.

In order to avoid random oscillations with frequencies determined not by the transmission line but by other *L-C* circuits occurring in the system, it is necessary that there should be no direct coupling between the anode and the grid circuits. Therefore the inductive coupling between the anode and grid coupling loops was made as weak as possible (see the next section) by giving these loops a suitable shape. Further, also capacitive coupling via the valve is prevented as far as possible by adequately earthing the grid of the valve, for high frequencies, with the large capacitance C_g (in which practically all self-inductance is avoided). Thus the excitation voltage is applied to the cathode, and it would be better to speak of a "cathode coupling loop". The grid bias required is taken from a supply unit with safety device; as soon as the oscillations deviate from the right frequency the grid bias is increased to such an extent that the valve is cut off.

The oscillatory condition of the circuit reads, to a first approximation:

$$(M_1 - M_2)\, M_2 \geq \frac{r}{\omega^2 S}\,, \quad . \ . \ . \ . \quad (2)$$

where M_1 and M_2 are the coefficients of mutual inductance between the coupling loops and the transmission line (fig. 2), r is the circuit resistance already mentioned, and S is the mutual conductance of the valve. Since ω is fixed and r and S cannot be chosen arbitrarily low and high respectively, it was necessary to give $(M_1 - M_2)M_2$ the highest possible value. Thus M_1 has to be as large as possible and $M_2 = \frac{1}{2}M_1$, which is actually the case. Since, however, M_1 and M_2 could only approximately be calculated in advance, it was again necessary to resort to the experimental method with a full-scale model to see whether $(M_1 - M_2)M_2$ would be large enough. By employing two transmitting valves of the type TA 12/20 connected in parallel, it was found possible to get a mutual conductance high enough to fulfil amply the oscillatory condition.

Set-up of the oscillator

Fig. 3 gives a rather more complete and more realistic but geometrically not yet quite exact representation of the oscillator. The two parallel-connected, water-cooled, transmitting valves (here only one is drawn) are mounted in an earthed metal casing divided, by a horizontal partition at grid level, into two entirely separated parts, the anode space and the cathode space. The anode coupling loop, a hollow copper conductor with two right-angled bends, passes out of the casing at the side (in the photograph in *fig. 4* it can just be seen) and then runs partly inside a channel of the inner conductor of the transmission line. In this way the desired tight coupling M_1 is obtained, since practically all the magnetic lines of force enveloping one conductor now envelop in this part of the loop also the other conductor, while at the same time the lines of force are kept as short as possible. At its free end the anode loop is earthed for high frequencies by means of a circle of vacuum capacitors (C_a in figs 2 and 3) which introduce hardly any self-inductance and have a very small loss angle, whilst being sufficiently proof against breakdowns to withstand the anode voltage of 12 kV; they can be seen in fig. 4 on the left. The cooling water for the anode is passed in and returned at this point through insulating hoses in the hollow conductor of the loop.

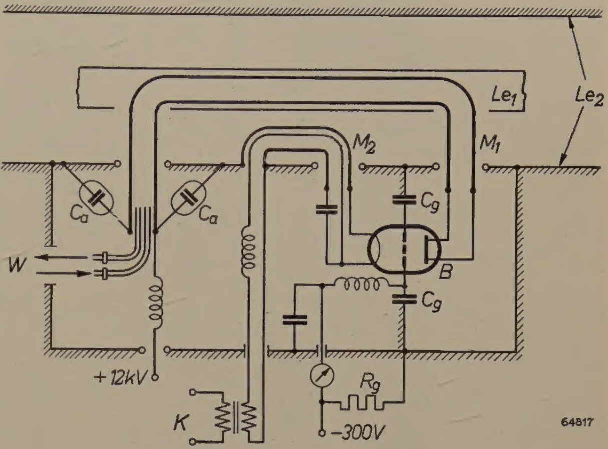

Fig. 3. Detailed circuit of the oscillator with the anode coupling loop M_1 and the cathode coupling loop M_2. For *B* two transmitting valves, type TA 12/20, connected in parallel are used. C_g capacitors used for by-passing the grid of each valve to earth, C_a capacitors for by-passing the anode coupling loop to earth, R_g grid resistor, *K* supply for the cathodes, *W* cooling water for the anodes.

The cathode coupling loop, a copper pipe somewhat shorter and thinner than the anode coupling loop, is led out of the earthed casing and approaches

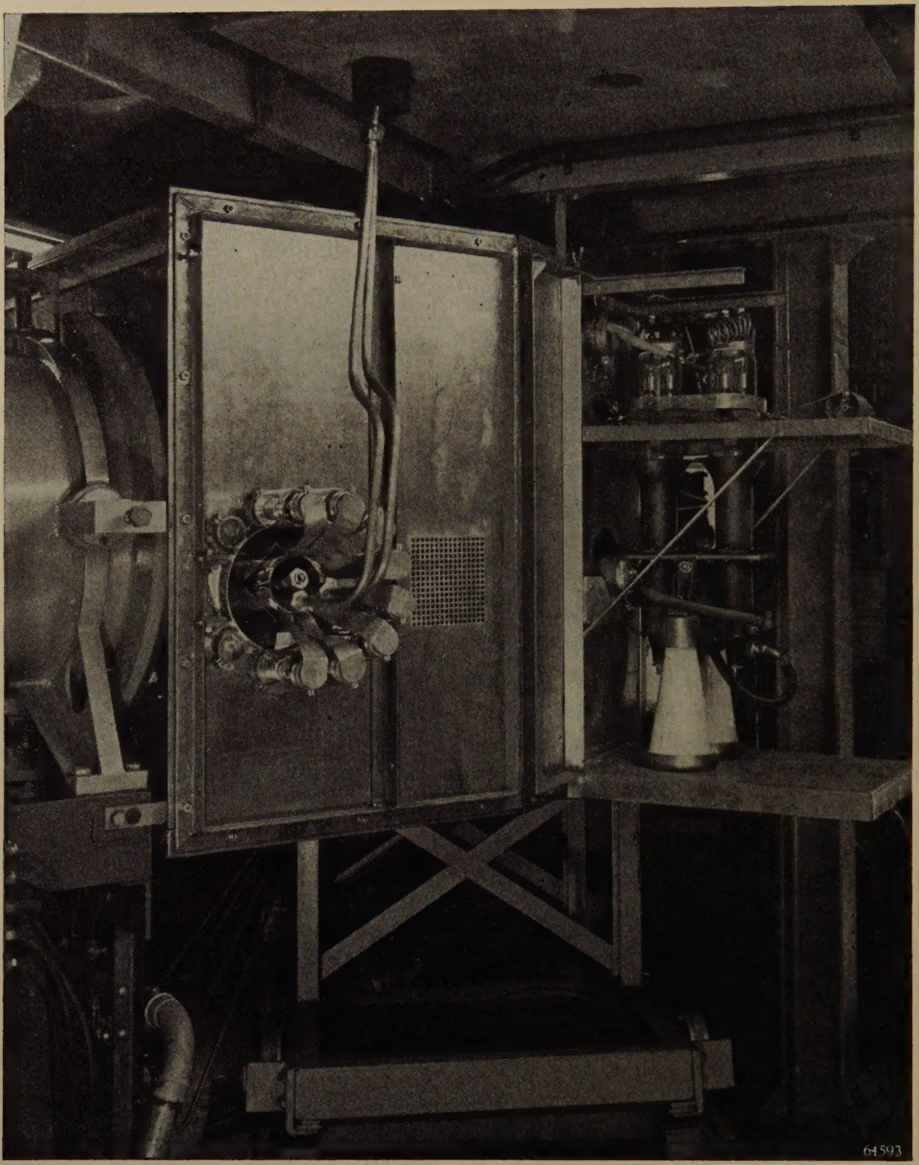

Fig. 4. The central part of the oscillator, opened. The two transmitting valves are seen with the anodes downward. The two hoses for the supply and return of the cooling water pass through the hollow anode coupling loop led through the wall on the left of the valves. Farther to the left is the end of the anode coupling loop connected to the earthed casing by a circle of vacuum capacitors. The connection is made with wide strips so as to keep the inductance low. Also the connection to the water supply pipes (disappearing at the top) is clearly seen. In the lower half of the photograph the undercarriage can be seen that supports half of the transmission line with the dee and runs on rails, so that these parts can easily be run out for any servicing work.

the inner conductor from the top. This could not be drawn in fig. 3, but it is illustrated in the sketch *fig. 5* and can partly be seen in the photograph of *fig. 6*. In this way the coupling between anode and cathode coupling loops is kept small. With the same object in view, the legs of the loops, which contribute towards the mutual coupling but not to the coupling with the transmission line, have been made as short as possible. The useful part of the cathode loop is about half the length of the anode loop, in accordance with the requirement $M_2 = \frac{1}{2}M_1$. The free end of the cathode loop, returning upward, is earthed, and at this point the filament voltage for the cathode is carried through the hollow conductor of the loop.

The grid of each transmitting valve is by-passed to earth by a circle of capacitors (C_g in figs. 2 and 3); here simple mica capacitors could be used, because the requirements as to the breakdown voltage are not so severe.

Although, as stated, the oscillatory condition was amply fulfilled, in the beginning difficulties were still encountered in the starting of the oscillator. The cause was found to lie in the oscillatory circuit being overloaded by discharge currents flowing in the accelerating chamber from the dee to earth and by electrons reaching the dee from the ion source. By applying a negative direct voltage of some hundreds of volts to the dee, via a choke and the inner conductor of the transmission line (the choke may be seen in the middle of fig. 6), the extra load was sufficiently reduced for the system to be easily brought into oscillation.

It was then found possible to generate an alternating voltage up to 25 kV_{peak} on the dee without any trouble from disruptive discharges in the gas, overheating of parts or overloading of the valves. Already with 5 kV_{peak}, however, the apparatus worked satisfactorily, yielding a reasonable beam of deuterons of 28 MeV. With the normal working voltage of 14 kV_{peak} on the dee the measured input power to the two valves was 5.7 kW and the anode dissipation 2.2 kW, so that the high-frequency output was 3.5 kW and the efficiency of the valves 61%. The value of the quality factor Q calculated from this output and the given values of V, ω and C (see equation (1)) agrees well with the value determined from the width of the resonance curve of the oscillatory circuit. The voltage on the dee is measured by means of a diode voltmeter and a capacitive voltage divider (P in fig. 1).

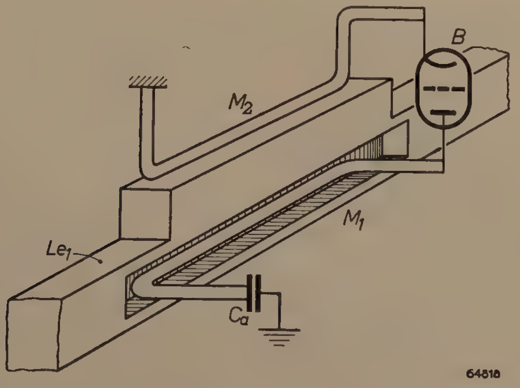

Fig. 5. Configuration of the anode coupling loop (M_1) and the cathode coupling loop (M_2). The inner conductor Le_1 of the coaxial transmission line has a channel in which the anode coupling loop is laid and an elevation where the cathode coupling loop is brought down close to the line. With this construction the mutual coupling between the two loops is kept as small as possible.

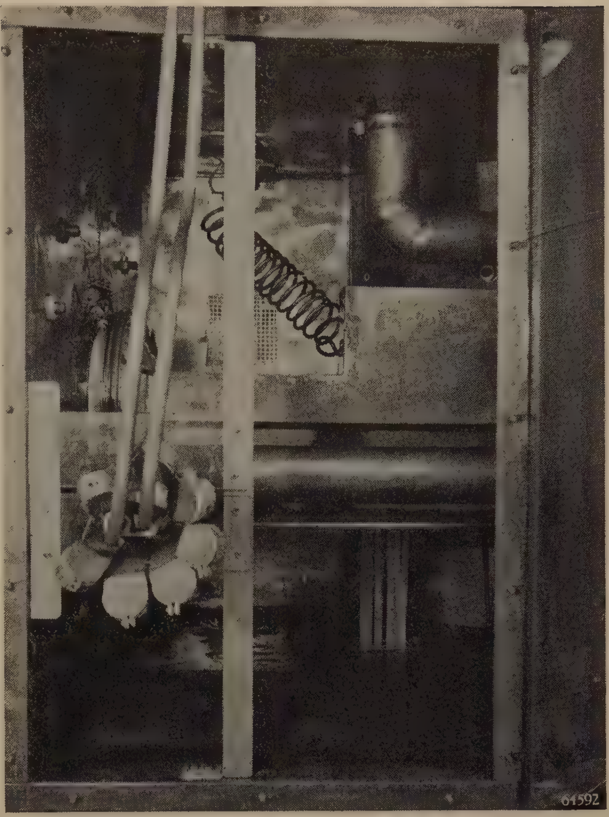

Fig. 6. View inside the middle part of the coaxial transmission line, showing part of the anode coupling loop (at the bottom) and part of the cathode coupling loop (top right). At the bottom on the left is the end of the anode coupling loop with connections for the circle of earthing capacitors and with water pipes.

The modulator

It has already been mentioned that the required 4% periodical variation of the oscillator frequency is brought about by a variation of the capacitance C_2 (fig. 1). Since approximately all the capacitance of the oscillatory circuit is lumped in C_1 and C_2, the total circuit capacitance is given by:

$$C = \frac{C_1 C_2}{C_1 + C_2}.$$

Since in our case $C_2 = C_1$ it follows that

$$\frac{\Delta C}{C} = \tfrac{1}{2} \frac{\Delta C_2}{C_2},$$

whilst further

$$\frac{\Delta \omega}{\omega} = -\tfrac{1}{2} \frac{\Delta C}{C},$$

so we find for the required relative variation of C_2:

$$\frac{\Delta C_2}{C_2} = -4 \frac{\Delta \omega}{\omega}.$$

Therefore, to reduce ω by 4%, we have to increase C_2 by 16%, i.e., in the case where $C_2 = 400$ pF, by 64 pF.

This capacitance variation is effected with the aid of a capacitor consisting of a toothed disc rotating between two sets of fixed vanes; see *fig. 7*. When the teeth of the disc are exactly opposite the vanes the capacitance is maximum, and when they are opposite the spaces between the vanes the capacitance is minimum. The disc has 40 teeth and makes 50 revolutions per second, so that the modulating frequency of the dee voltage is 2000 c/s.

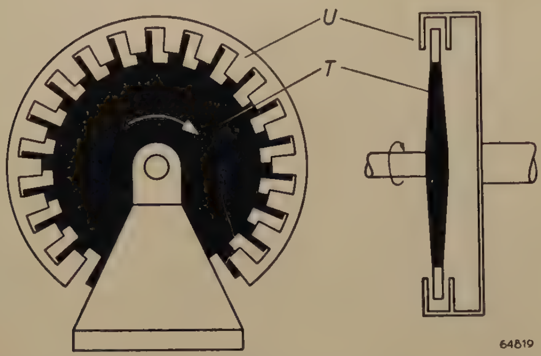

Fig. 7. The modulator comprises a periodically varying capacitance obtained with a toothed disc T rotating between two sets of fixed vanes U. The difference between maximum and minimum capacitance is 64 pF.

Connected parallel to this capacitor is a second one with a constant capacitance, such that the total capacitance in the minimum position of the rotating capacitor is approximately the desired value $C_2 =$ 400 pF. This fixed capacitor can be slightly adjusted with the aid of a servomotor, during operation, to get exact resonance between the frequency of the oscillator and the travelling particles in the accelerating chamber. Resonance can be judged by the occurrence of a maximum in the current flowing to the target, or by the concentration of neutrons around the accelerating chamber.

Both the rotating capacitor and the fixed capacitor are mounted in an evacuated drum (10^{-5} mm Hg), thus making it possible for the clearances between the rotating and the fixed capacitor parts to be kept relatively small (6 mm) in spite of the high voltages prevailing, and thus to obtain the required large capacitances with a comparatively small plate surface. In this way it has been possible to keep the modulator small enough to be able to work with the high speed of 3000 r.p.m. and thus reach the high modulating frequency of 2000 c/s 1)).

From the circuit diagram of *fig. 8* it is seen that one plate of the fixed capacitor (C'') and the fixed vanes of the variable capacitor (C') are connected to the inner conductor of the transmission line. The other capacitor plate and the rotating toothed disc have to be earthed. The earthing of the rotating disc could be achieved, in principle, by means of a sliding contact (brushes), but this was found to be unsatisfactory owing to the too high brush resistance for high-frequency currents and on account of arcing arising from the sparking in vacuo. The disc is therefore earthed by means of a large capacitor consisting of a number of fixed, earthed plates and, rotating between them (with a spacing of 2 mm), a number of plates mounted on and electrically connected to the spindle of the rotating capacitor disc C'. The earthing of C' obtained in this way is not, it is true, complete, since the capacitances of C' in series with those of the earthing capacitor form a voltage divider, so that the rotating disc and the rotating plates of the earthing capacitor continue to be at a small high-frequency voltage, of about 0.5 kV. This necessitated insulation of the rotating part of the modulator in the manner indicated in *fig. 9*. The disc and the plates are mounted on a bush turning in two ball bearings about a fixed, water-cooled, shaft insulated with blocks of a ceramic insulating material ("Kersima"). The set-up is such that the capacitance between the shaft and earth is small, so as to avoid any appreciable high-frequency currents flowing across the ball bearings.

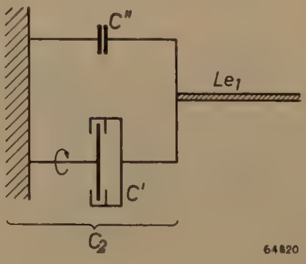

Fig. 8. Circuit of the modulator. The periodically varying capacitor C' is shunted by a constant capacitor C'' for making up the desired total capacitance. The rotating disc of C' has to be earthed.

These currents could be further reduced in a well-known way, by shunting the capacitance just mentioned by a coil to form a circuit tuned to 10.7 Mc/s, which thus has a maximum impedance for that frequency.

The rotating part of the modulator is driven by a motor also placed in vacuo. It would have been possible to drive the modulator from outside the

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1) In this connection it is important that a capacitance variation of only 64 pF is needed. If the frequency sweep has to be much greater than 4% (thus for a higher final energy of the particles, and for protons instead of deuterons; see I), then rotating capacitors of a much higher value are needed, and it is then not so easy to reach high modulating frequencies.

vacuum drum but then it would have meant having to pass through the wall of the drum a motor shaft continuously rotating at a speed of 3000 r.p.m. while still maintaining a vacuum of 10^{-5} mm Hg, which would not have been so easy to ensure. Our solution is very simple, only the three-phase voltage supply cables for the motor having now to be led in through the wall of the drum. The rotor of the induction motor has the form of an iron bush slipped onto the bush mentioned above, which carries the rotating parts of the modulator. The three-phase stator producing the rotating field, is placed around the bush with a gap of a few mm in between. An essential condition is that in vacuo the motor must not give off too much gas or impurities, and this condition is very difficult to fulfil with field coils of insulated wire. The motor has therefore been built for a very strong current (each of the three phases 185 A, voltage 2.1 V; the starting current is 300 A) and a very small number of windings. Thus the windings could be made in the form of free, bare copper tubes through which cooling water can flow and which are mounted on two "Pertinax" plates (the only insulating material contained in the motor). This and other details of the construction are shown in *fig. 10.*

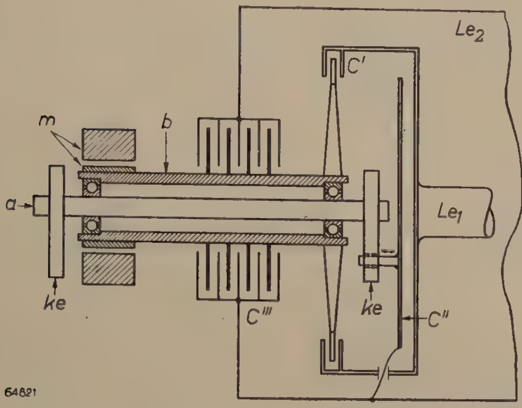

Fig. 9. Construction of the modulator, with the rotating capacitor C', the fixed (adjustable) capacitor C'' and the earthing capacitor C''' (schematically drawn). Le_1 and Le_2 inner and outer conductors of the transmission line; *b* bush carrying the rotating capacitor parts and rotating about the shaft *a* insulated with two sets of "Kersima" blocks *ke*; also mounted on *b* is the rotor (an iron bush) of the induction motor *m*.

As the motor has only a small torque and the system has to start quickly in order to pass smoothly over some critical speeds, the bearings of the system have been very carefully designed. The friction is so small that it takes more than an hour for the system to run out to a standstill (it is to be remembered that there is no air resistance). The bearings can be lubricated from the outside with special vacuum oil, but this proves to be hardly ever necessary.

It has been mentioned that the dimensions of the parts of the oscillator, including the modulator, had to be determined experimentally with models in order to make sure of getting the right frequency. Measurements taken show that as a result the frequency fluctuates between 10.46 and 10.11 Mc/s. This is a variation of slightly less than 4%, but it proved to be more than sufficient. From the resonance condition $\omega = qB/m$ it follows that at the maximum frequency resonance occurs when $B = 1.36$ Wb/m². The actual flux density in the middle of the accelerating chamber is a little more than this, so that resonance is not reached until the frequency has dropped slightly. Consequently during a cycle of the modulating frequency acceleration takes place roughly between the instants *M* and *N* in *fig. 11*, i.e. during somewhat less than half 1/2000 sec, thus about 1/5000 sec.

The shape of the frequency variation curve, which is related to the chosen form of the teeth and blades of the rotating capacitor, is not critical, owing to the phase stability of the travelling particles.

Here something has to be added about the modulating frequency. It has been pointed out that the high value of 2000 c/s is favourable for obtaining a high average beam current on the target. The

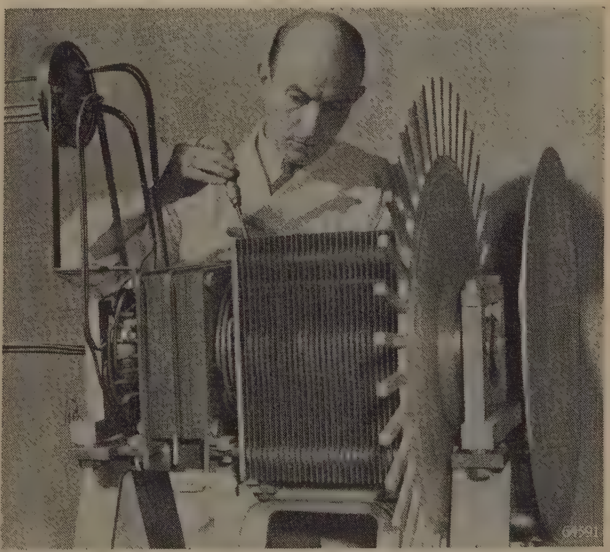

Fig. 10. The modulator (without the parts of the capacitors C' and C'' fixed to the transmission line; see fig. 9). From right to left: the slightly adjustable plate of C'' (guided by three pins), the toothed disc of C', the large earthing capacitor C''', and the stator coil of the driving motor with the peculiar "windings" for the three phases made up of copper water pipes standing free.

question arises whether a still higher modulating frequency would be desirable.

In our synchrocyclotron the deuterons have a peripheral rotational speed of roughly 10^7 "loops" per second (viz. about equal to the frequency of 10.7 Mc/s of the voltage on the dees). The acceleration of a particle to the final energy of 28 MeV has to take place in about 1/5000 sec, in which time the particle describes $10^7/5000 = 2000$ loops, so that it is accelerated by the dee voltage 4000 times and has to gain 7 keV in energy each time. At a peak value of 14 kV of the sinusoidally alternating dee voltage, the particles coming under the phase-stabilizing action and accelerated to the final energy are those which pass the gap with a phase angle $\varphi = \text{arc cos } 7/14 \approx 60°$ with respect to the peak of the dee voltage (see A). If a higher modulating frequency is chosen then a smaller angle φ is needed to allow the particle to reach the final energy of 28 MeV in the shorter time available. This, however, makes the orbit less stable, the phase tolerances become smaller and there are fewer ions in a group. Consequently the mean beam current will increase less than proportionately with the number of current surges per second and ultimately even decrease.

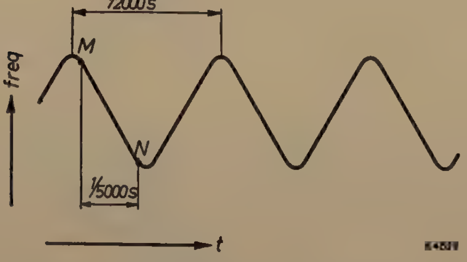

Fig. 11. Variation of the oscillator frequency with time. Modulation frequency 2000 c/s. Acceleration of the deuterons takes place during the interval MN of each modulation cycle.

Indeed it is seen that during the starting period of the modulator the mean current on the target increases first roughly in proportion to the modulation frequency and then more slowly. At 2000 c/s (and 14 kV dee voltage) the expected maximum is not yet reached. Constructional difficulties have for the time being prevented any further increase of the modulation frequency. It is particularly the mechanical stresses in the rotating toothed disc that then constitute a serious problem: at a peripheral speed of 110 m/sec the material has to withstand very great forces. That is also why, instead of copper, duraluminium was chosen for this disc.

We can now also understand the influence of the dee voltage on the choice of modulation frequency as already referred to in passing. If instead of 7 kV the gain in energy per "loop" were to be only 3 kV then the acceleration to the final energy would take $^7/_3$ times as long and the modulation frequency (for the same φ) could not be more than 850 c/s. The Harwell synchrocyclotron does in fact work with 3 kV gain per loop, so that for a final energy of 180 MeV the particles have to make 30,000 loops, and, as the mean oscillator frequency of that cyclotron is about 23 Mc/s, the modulation frequency cannot be made any higher than about 300 c/s.

Set-up of the transmission line with modulator and dee

Taking a final look over the whole assembly of oscillatory circuit with dee and modulator, we find there are still some other constructional problems typical for the synchrocyclotron.

The inner conductor of the transmission line must be insulated. In order to avoid excessive dielectric losses in the supporting insulators the latter have preferably to be placed where no high-frequency voltages occur. As we have seen in fig. 1, this is the case near the middle of the line. Thus, roughly speaking, we get the picture of a copper pipe about 15 cm in diameter and about 5 metres long supported only over a short distance near the middle, carrying at one end the weight of the dee and at the other end the vanes of the variable capacitor between which the toothed disc is rotating at a speed of 3000 r.p.m.

Since the copper pipe would not be able to bear this mechanical load it has been provided with an internal frame of brass.

Another complication is the fact that the pipe, at least at the ends, has to be in vacuo. The simplest solution would appear to be an arrangement whereby the whole transmission line is in vacuo, thus joining up the modulator drum and the accelerating chamber into one continuous vacuum. In that case the ports through which the coupling loops of the oscillator are passed would have to be vacuum-tight, whilst in the event of any changes having to be made to these couplings or in the rather complicated construction of the modulator the vacuum of the whole cyclotron would of necessity be disturbed and have to be restored. We therefore preferred to separate the vacuum of the accelerating chamber and the modulator and to give each its own pumping system (see I, fig. 1). The two oscillator coupling loops are now easily accessible and come to lie in air. On either side of the anode coupling loop are the supporting insulators carrying the inner conductor of the transmission line.

Since this conductor has to be led into the two vacuum chambers via vacuum-tight insulators it would have been obvious to use the supporting insulators for this purpose. In fact, also the port insulators must not be too far away from the voltage node. Accordingly, the vacuum chamber of the modulator and also that of the accelerating chamber is in fact extended close to the voltage node by a cylinder enveloping the copper outer conductor of the line. Nevertheless it was found necessary to keep the functions of supporting and port insulators separate. These insulators already have to answer rather exceptional requirements: the dielectric losses at a frequency of 10 Mc/s have to be low, in connection with which also the dielectric constant has to be small, and they must stand up to a voltage of at least several kilovolts (they are still that far away from the actual voltage node). Furthermore, the port insulators have to make a vacuum-tight seal of rather large dimensions (the pipe to be passed through is 16 cm in diameter), and the supporting insulators have to be strong enough to bear the weight of the pipe with dee and modulator vanes. Only with the functions separated was there enough freedom in the shaping of the insulators for answering the requirements in both cases. As insulating material "Kersima" [2] was again used, a ceramic material with very good electrical and mechanical properties, in particular exceptionally high compressive strength (porcelain would be quite unsuitable). *Fig. 12* shows the ultimate set-up.

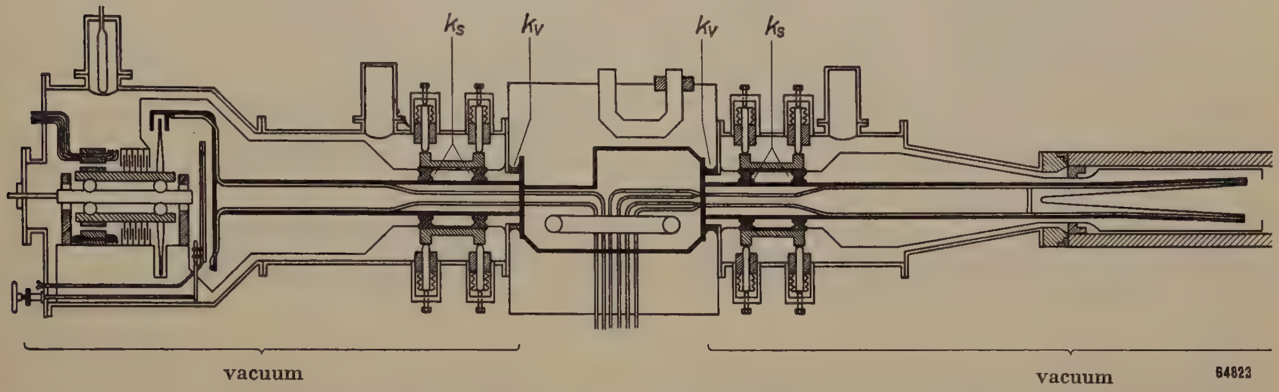

Fig. 12. Set-up of the oscillatory circuit. The approx. 5 m long copper inner conductor of the transmission line, with at the ends the dee and the vanes of the modulator (all drawn in heavy lines), is supported by the ceramic insulators k_s ,which, in order to reduce their electric load, are placed not far from the voltage node in the middle of the line. To give the wide copper pipe the necessary strength it is fitted with a brass frame on the inside (not show in the drawing). For centering the system the supporting insulators can be shifted by means of screws (and vacuum-tight metal bellows). The dee and the modulator are each in separate vacuum chambers, so that the middle part of the line with the coupling loops is readily accessible. The ceramic insulators k_v provide for vacuum-tight ports through which the line can be passed. Also part of the cooling water pipes is shown in the drawing.

To complete the picture it is to be mentioned that in the conductors of the transmission line, with dee and modulator, with high-frequency currents of about 250 A_{rms} in permanent working a power of over 3 kW is dissipated. Owing to the insulated mounting and the fact that the greater part of the system is in vacuo, only a small part of this heat can be carried off by conductance or by convection. Neither can radiation help much in cooling, since owing to the high frequency the surface cannot have a properly radiating layer but rather has to be bright. This accounts for the extensive use of water cooling, which to the uninitiated may at first sight be astonishing. Water pipes run to all parts of the transmission line, the dee and the modulator. Of course the cooling water inlet is in the middle of the line, at the point of the voltage node. Since, however, in connection with the frequency modulation this node swings somewhat to and fro, the pipe system is connected to the municipal water mains with insulating hoses of polystyrene rubber (see fig. 6 at the bottom on the right), which have only very low dielectric losses.

Finally, by way of demonstrating with an example the very close relationship of all constructional details of the synchroncyclotron, it may be pointed out that it is in fact due to the properties of the ceramic insulating material that two transmitting valves are needed in the oscillator: if the properties of the insulating material were such that the supporting and port insulators could be placed farther away from the middle then more space would be available for the anode coupling loop, higher values of the inductances M_1 and M_2 could be reached and the oscillatory condition (2) could be fulfilled with a smaller mutual conductance, thus with only one valve.

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[2] Cf. Philips Techn. Rev. 10, 205, 1948.

Summary. I. The Philips synchrocyclotron at Amsterdam is producing in continuous operation deuterons with an energy of 28 million electronvolts, which are being used for many forms of nuclear transmutations. The fundamental considerations to be met in the designing of the installation were, apart from the choice of the particles to be accelerated and the final energy desired, that the beam current should be as high as possible and the working as reliable as possible. At this moment a current of 20 microamperes is already being reached with the energy mentioned. This energy of 28 MeV may just lie within the scope of the classical cyclotron, but the application of the principle of the synchrocyclotron makes the construction very much simpler and the installation more economical in working. In the general description the most important parts of the installation are mentioned and an idea is given of the extent of the accessory apparatus required.

II. In a synchrocyclotron the particles to be accelerated can without objection be made to describe a large number of loops, so that in this case a dee voltage of only about 15 kV_{peak} is needed. For resonance with the travelling particles the frequency of this voltage has to be 10.7 Mc/s (wavelength about 30 m). The oscillatory circuit in which this frequency is generated consists of a coaxial transmission line with at each end a high capacitance (about 400 pF) formed by one of the dees (the other dee is earthed) and the modulator respectively. Owing to these large capacitances, high-frequency currents of about 350 A (peak value) are flowing in the system, but by very careful construction the dissipative resistance has been limited to about 0.05 ohm, so that the dissipation is no more than about 3 kW. Of this about 560 watts is dissipated as "useful energy" by the beam of deuterons on the target. All parts of the oscillatory circuit are cooled with water. The transmission line is made to oscillate by means of a Meissner circuit, comprising two TA 12/20 transmitting valves connected in parallel. The mode of vibration is such that a voltage node occurs in the middle of the line. Parasitic oscillations are suppressed by by-passing the grid of the valves to earth and giving the anode and cathode coupling loops a suitable configuration. The frequency is periodically varied by 4% by means of the modulator at the free end of the transmission line; the modulator comprises a capacitor with a rotating toothed disc in vacuo. The driving motor is likewise in vacuo, this being made possible by a special construction (few windings, field current 185 A, voltage 2.1 V). The modulating frequency is very high, viz. 2000 c/s. The vacuum spaces of modulator and accelerating chamber are separated. The insulated, about 15 cm thick, inner conductor of the transmission line is led into these spaces via port insulators made of "Kersima", which material is likewise used for the supporting insulators bearing the inner conductor, the (non-earthed) dee and part of the modulator. The supporting and the port insulators are situated close to the anode coupling loop, where the high-frequency voltage is low.

THE ELECTRO-ANALOGUE, AN APPARATUS FOR STUDYING REGULATING SYSTEMS

I. COMPONENTS AND FUNCTIONS

by J. M. L. JANSSEN and L. ENSING.

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Mathematical operations such as adding, subtracting, multiplying, dividing, differentiating and integrating can be carried out with the aid of electrical circuit elements. On this principle it is possible to construct electrical models of, say, the processes of heat exchange or of mechanical processes. Such models are attractive because of the ease with which, by employing an oscilloscope, a sufficiently accurate picture can be obtained of the working of the process imitated, as also of the effect produced when various parameters are altered. Electrical models have proved their value particularly in the case of automatic controllers used to keep a certain working factor constant. An electro-analogue is a collection of instruments required for building up an electrical model for such cases and studying its behaviour. This first article will deal with the components required for building an electro-analogue. Details of the electrical circuits will be discussed in another article to follow.

In industry nowadays automatic controllers are being employed on a large scale for stabilizing certain quantities — temperature, voltage, current, resistance, acidity, rate of flow of a liquid or gas, etc. Some considerations were given to this subject in a previous article in this journal [1]), where it was pointed out that in a particular case, for designing the best regulating system or for the best manner of adjusting a particular automatic controller, the purely theoretical process of the calculation cannot be followed: the mathematical difficulties in taking into account all the pertinent factors would be far too great even in fairly simple cases. Neither is it advisable, in many cases, to follow entirely empirical methods, since this may lead to lengthy interruptions in the industrial process, with the resultant decline in the quantity and quality of the production.

As already mentioned in the article quoted, a solution can often be found by working with an electrical model of the process to be controlled and of the controller itself, thereby choosing a time scale such that the phenomena can be observed on the screen of an oscilloscope. Measurements of the actual process can be limited to the recording of the step-function response, i.e. the characteristic indicating how the quantity to be regulated responds as a function of time to a sudden change of the regulating quantity at the input of the process.

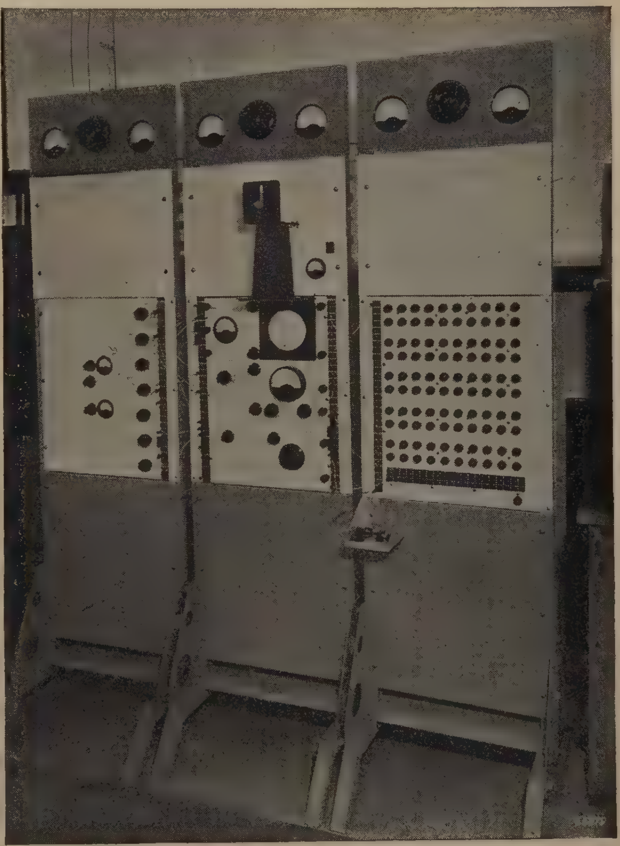

Fig. 1. Photograph showing the electro-analogue. The left-hand panel comprises mainly the model of the automatic controller, the middle one the oscilloscope and accessories, and the right-hand one the model of the process. At the top of each rack is a variable transformer for adjusting the A.C. voltage supply to the correct value, shown by the voltmeter on the right, and an ammeter (on the left) indicating the total current consumption of the panel.

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[1]) H. J. Roosdorp, Philips Techn. Rev. **12**, 221-227, 1950/51 (No. 8).

The electrical model of the process has to be of such a construction as to show the same step-function response.

Given an apparatus which is designed for building up such an analogous model of the process and which comprises, further, a model of the automatic controller in which a proportional, an integrating and a differentiating term can be made to play a part as needed, and which moreover comprises an oscilloscope, a square-wave generator and further accessories, then the effect of altering various parameters in the controller, of introducing disturbances at various points, etc. can be seen at a glance. Such an apparatus is the "electro-analogue", a specific design of which is illustrated in *fig. 1.*

In addition to solving concrete regulating problems the electro-analogue has proved its worth in research work in the field of regulating. The great variety of models — both linear and non-linear — that can be built with it and the clarity of the results have contributed much towards a broader insight into regulating problems.

We shall first describe the components of which the electro-analogue built by us consists, while the electrical execution of the most interesting parts will be discussed in another article.

Process analogue

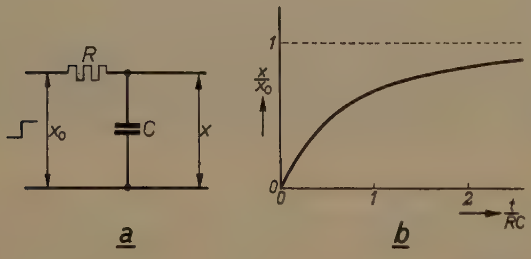
Analogous networks for some idealized processes

There has been no lack of attempts to put the step-function response of a more or less idealized process into mathematical form and then to translate that form into an electric network serving as a "process analogue", i.e. as an electrical model of the process.

Many processes are mainly characterized by an inertia caused, for instance, by a thermal resistance and a thermal capacity. Obviously in such cases the process analogue can to a first approximation be composed of a resistance R and a capacitance C in series (*fig. 2a*). The equation for the step-function response of this circuit (fig. 2*b*) is:

$$\frac{x}{x_0} = 1 - e^{-\frac{t}{RC}}.$$

where x_0 is the amplitude of the applied voltage step, x is the voltage across the capacitor and t is the time.



a b

Fig. 2. *a*) Resistor R and capacitor C in series, as the simplest electrical model of a process with thermal resistance and thermal capacity. *b*) Step-function response curve of this network.

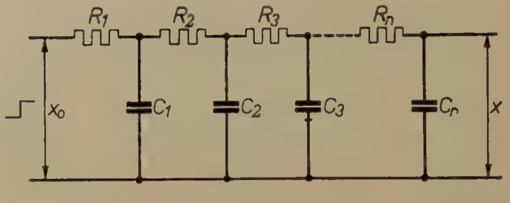


Fig. 3. Extension of the network in fig. 2*a* to a series of n R-C sections connected in cascade.

Usually, however, one has to do with a series of thermal (or other) resistances and capacities. A circuit according to *fig. 3* then gives a better approximation of the situation. In this network the sections R_2-C_2, R_3-C_3 etc. constitute a certain load on all the preceding sections, as is more or less the case in an actual process.

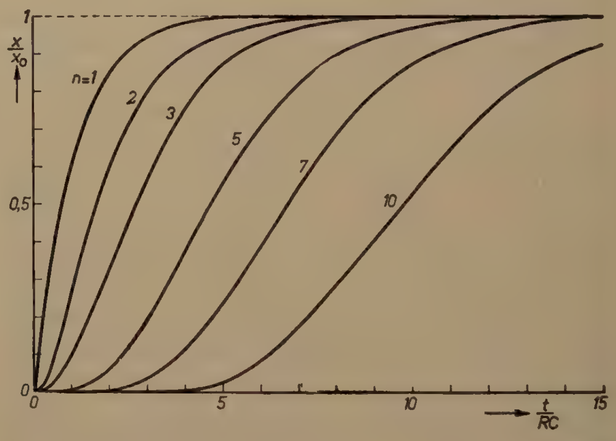


Fig. 4. Calculated step-function response curves for n = 1 to 10 sections of the network according to fig. 2 with equal resistors R and capacitors C in all sections, these sections being separated by interposed amplifying valves so that the one section does not constitute a load on the preceding sections.

Other hypothetical processes in combination with an automatic controller have been theoretically studied by mathematicians of the "Bataafsche Petroleum Maatschappij" [2]): one of these processes has its electrical analogue in a series of R-C sections where there is no such loading of the preceding sections by the following ones as just referred to.

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[2]) P. Hazebroek and B. L. van der Waerden, The optimum adjustment of regulators, Trans. Amer. Soc. Mech. Engrs. 72, 317-322, 1950 (No. 3).

This case can be realized by separating the sections in the network of fig. 3 by means of amplifying valves (details will be given in a subsequent article). The calculated step-function response curves of such a network for different numbers of sections are given in *fig. 4*.

To verify the proper functioning of the electro-analogue this is provided with both kinds of *R-C* networks. The network without interposed valves consists of 20 "coupled", and that with valves of 40 "decoupled" identical sections, any number of which can be used as required. *Fig. 5* shows the oscillograms of the step-function responses of 10 sections of both networks.

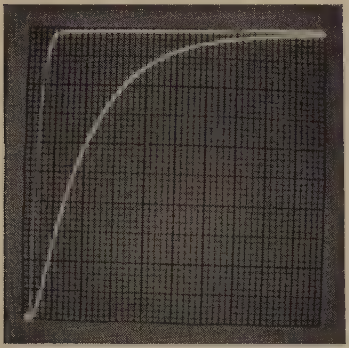

Fig. 5. Oscillograms of the step-function response curves of 10 sections of an *R-C* network; upper curve with decoupled sections, lower curve with the sections not decoupled. The upper curve agrees with the calculated curve for $n = 10$ in fig. 4, in which figure a widely different time scale has been used.

These networks have rendered good service particularly for research purposes, but they do not form a universal process analogue with which any step-function response curve can be obtained. To arrive at such a universal process analogue, instead of adhering to an electrical analogy with the physical working of the actual process, an entirely different network has been devised by means of which any step-function response can be sufficiently approximated by a discontinuous line.

Universal process analogue

In the universal process analogue used by us an electric delay network [3]) is employed. This consists of a large number of sections, made up of capacitors and inductors, so arranged that voltages are transmitted, unattenuated, with a delay time τ which over a very wide frequency range is independent of the frequency. When a voltage V_0 having the shape of a step-function is applied to the input of the network then the output voltage V_1 of the first section likewise has the shape of a step-function but the step is delayed by a time τ (*fig. 6*) and is thus produced at $t_0 + \tau$; the voltage V_2 behind the second section, also with the shape of a step function, makes the step at $t_0 + 2\tau$, and so on.

If one now has a device with which any arbitrary proportions $a_1, a_2, a_3 \ldots$ of the voltages V_0, V_1 $V_2 \ldots$ can be added — which proportions it must be possible to adjust independently of each other between the limits $+a_{\max}$ and $-a_{\max}$ — then one can obtain a sum voltage V_s:

$$V_s = a_0 V_0 + a_1 V_1 + a_2 V_2 + \ldots,$$

of echelon shape as shown in *fig. 7*, and with this

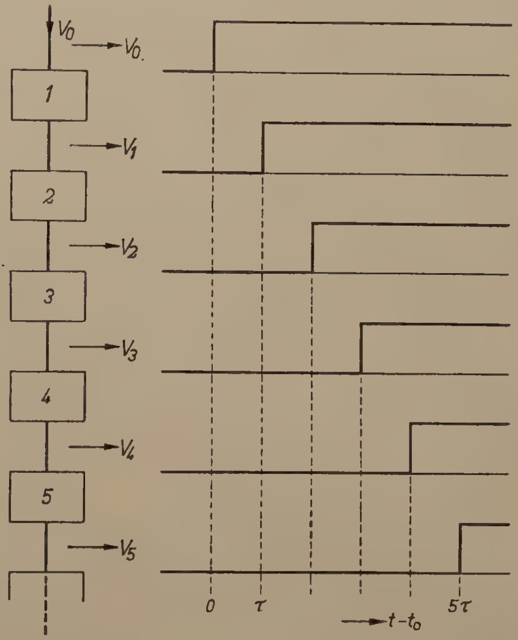

Fig. 6. Left: delay network with sections *1, 2, 3,* ... When a voltage step V_0 (see right-hand part of the diagram) is applied to the input then there arise at the outputs of the sections equally large pulses V_1, V_2, $V_3 \ldots$ lagging a time τ, 2τ, 3τ, ... behind V_0.

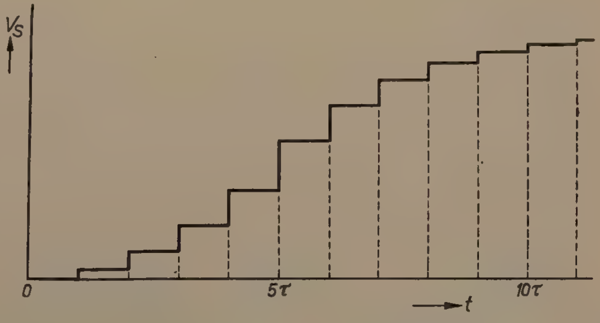

Fig. 7. When variable fractions a of the voltages V_0, V_1, V_2, V_3, ... of fig. 6 are added together, a sum voltage V_s of echelon shape is obtained, with which any curve can be approximated by varying the coefficients a.

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[3]) British patent specification No. 517,516 in the name of A. D. Blumlein, H. E. Kallmann and W. S. Percival.

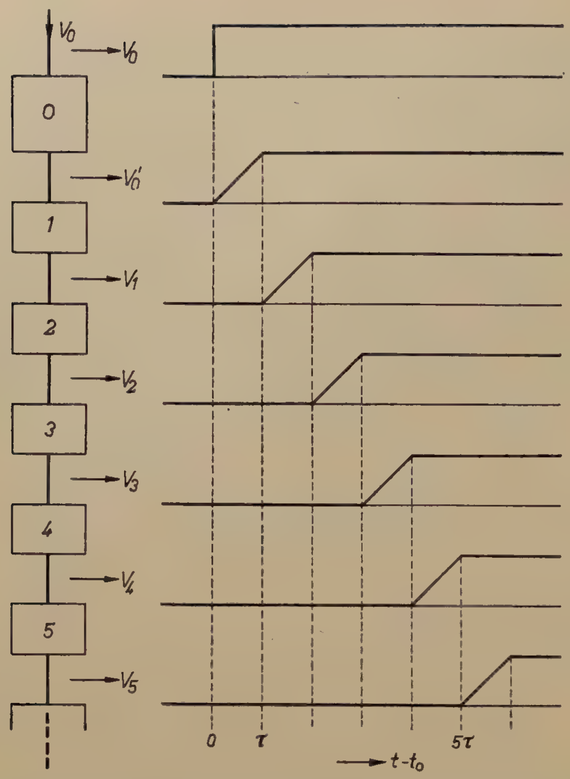

Fig. 8. As in fig. 6 but with the delay network preceded by a section *0* which converts the step voltage V_0 into a voltage V_0', which in the interval τ rises linearly with time.

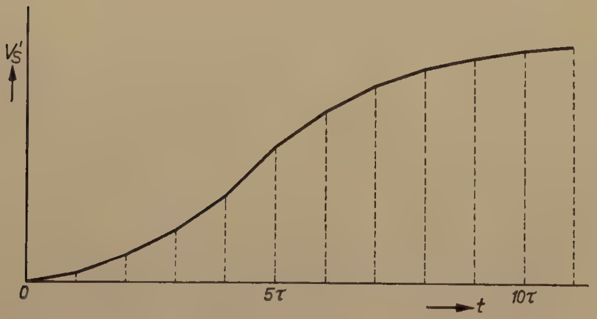

Fig. 9. Adding up variable fractions a of the voltages V_0', V_1, V_2, V_3, ... of fig. 8 yields a sum voltage V_s', the curve of which more closely approximates to a smooth line than the curve of fig. 7.

it is possible to approximate quite well the measured step-function response of any actual process.

The approximation is much better still when the voltage V_0, having the shape of a step function, is first converted into a voltage V_0' which in the interval τ increases, for example linearly, from 0 to the maximum value. This conversion can be brought about in a section (*0* in *fig. 8*) preceding the delay network. The shape of the voltages V_0', V_1, V_2 ... is then as shown on the right in fig. 8, and the sum voltage V_s' follows an almost smooth curve (*fig. 9*).

In our electro-analogue the method last outlined is applied. The delay network consists of 50 sections. There is no trace of any kinking in the curve of the sum voltage.

Oscilloscope and square-wave generator

For making the step-function response of the process analogue identical with that of the actual process it is necessary to be able to observe both these characteristics simultaneously. To that end the variation of the output voltage V_s' of the delay network is displayed on the screen of a cathode-ray oscilloscope, over which a transparent sheet of cellophane marked with a system of coordinates is placed, on which the measured step-function response of the actual process has been plotted. The curve of the process analogue is then matched as closely as possible to the plotted curve by turning the knobs with which the coefficients a_0, a_1 ... a_{50} are varied (see the right-hand rack in fig. 1).

For oscilloscopic observation it is advisable that the voltage V_0 does not make one single step but that this is repeated periodically, thus using a square-wave voltage (*fig. 10a*). This voltage is produced in a square-wave generator, in which the peaks of a sinusoidal voltage (50 c/s) taken from the mains are clipped off, leaving a practically pure square-wave voltage.

The output voltage V_s' of the process analogue now has a shape as represented in fig. 10*b*. The dotted part of this curve is made invisible by periodically blacking out the electron beam in the cathode-ray tube. The fully-drawn part is traced faithfully on the screen of the oscilloscope on account of the fact that in the respective half-cycles the time-base voltage is linear.

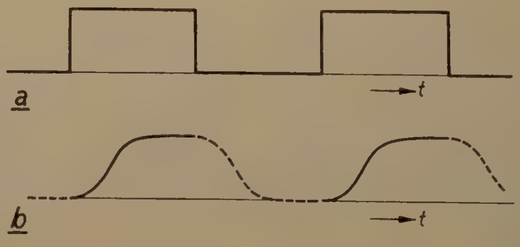

Fig. 10. *a*) Square-wave voltage at the input of the section *0* (fig. 8). *b*) Example of the variation of the sum voltage V_s' traced on the screen of an oscilloscope (the dotted part is blacked out).

Models of continuously acting automatic controllers

In the article quoted in footnote [1]) a distinction was made between discontinuously and continuously acting automatic controllers. We shall first consider only the latter type, leaving the other type of controllers to be dealt with in the last section of this article.

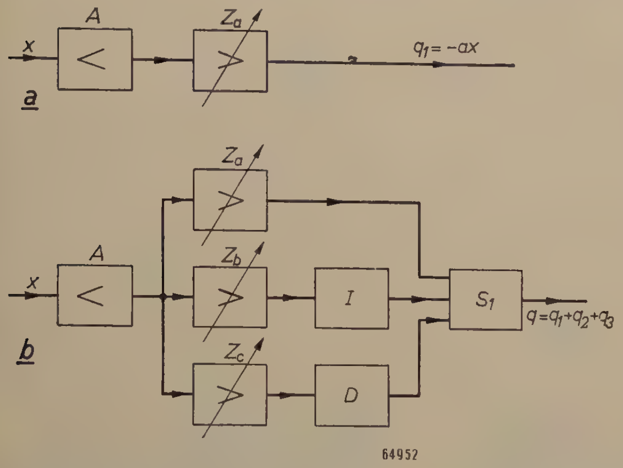

Fig. 11. *a*) Block diagram of a model of an automatic controller with proportional action: amplifier A and variable attenuator Z_a. *b*) Block diagram of the model of an automatic controller with proportional, integral and derivative action. A is a common amplifier, I the integrator, D the differentiator. The coefficients a, b and c of the three components are adjusted by means of the variable attenuators Z_a, Z_b and Z_c respectively. S_1 is a collecting stage in which the three components are added together.

In the equation for the relation existing between the amount x by which the quantity to be regulated deviates from the desired value and the amount q by which the regulating quantity is consequently changed by the automatic controller, various terms may occur, e.g. a proportional, an integrating and a differentiating term [4]):

$$q = -ax - b\int x\,\mathrm{d}t - c\frac{\mathrm{d}x}{\mathrm{d}t} = q_1 + q_2 + q_3. \quad (1)$$

It must be possible to realize these three terms in the electrical model of the automatic controller and the coefficients a, b and c have to be variable in the model.

The proportional term, $q_1 = -a\,x$, is easily realized by means of an electronic amplifier. The coefficient a can be made variable by adding a variable attenuator (*fig. 11a*).

The integrating term, $q_2 = -b\int x\,\mathrm{d}t$, and the differentitating term, $q_3 = -c\,\mathrm{d}x/\mathrm{d}t$, are obtained with the aid of an integrating and a differentiating network respectively. Separate attenuators provide for the variability of the coefficients b and c (fig. 11*b*). Finally the three voltages, q_1, q_2 and q_3, are summated in a "collecting stage".

The electro-analogue is provided with a second integrator and a second differentiator by means of which terms of the type $b_2\int\int x\,\mathrm{d}t\,\mathrm{d}t$ and $c_2\,\mathrm{d}^2x/\mathrm{d}t^2$ can also be added, though as a rule there is no need for this.

Form and location of the disturbances

When a process analogue has been built up in the manner described then this network is combined with the model of the automatic controller according to fig. 11 *b* to form a closed circuit (*fig. 12*). Contrary to fig. 11*b*, a common attenuator Z_0 is added with which the coefficients a, b and c are varied simultaneously.

The oscilloscope can be connected to various points of the circuit by means of a lead. When it is found that the situation is stable — this can always be reached by adjusting Z_0 for sufficient attenuation — then at all input and output terminals of the component parts of the regulating circuit the voltage will be zero. Upon a disturbance being introduced somewhere, then at the output of the process analogue and at that of the model of the automatic controller voltages will arise which are proportional to the quantities x or q respectively, and on the screen of the oscilloscope it can be seen how these voltages vary with different values of the coefficients a, b and c.

The disturbances occurring in a regulating system in practice can be distinguished both ac-

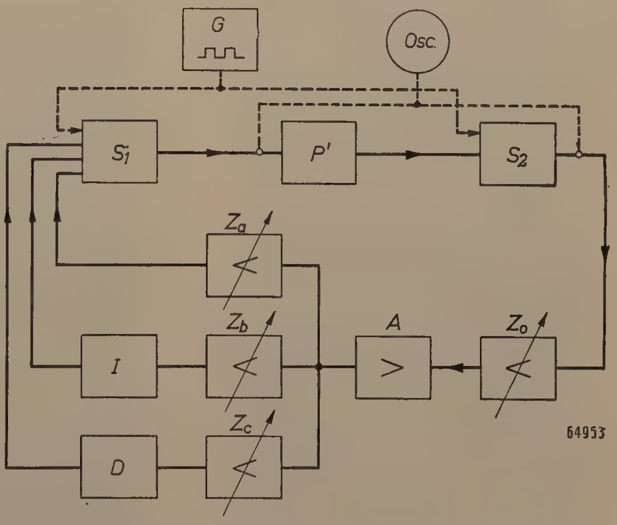

Fig. 12. Block diagram of the model of a regulating circuit. P' process analogue, whose step-function response has been matched to that of an actual process. Z_0, A, Z_a, Z_b, Z_c, I, D and S_1 form the model of the automatic controller (fig. 11). G square-wave generator supplying a disturbance, which can be applied, for instance, to the input of the process (at S_1) or at its output (at the collecting stage S_2 provided for that purpose). The oscilloscope Osc can be connected to various points of the circuit. Z_0 is a common attenuator.

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[4]) In order to get positive numerical values of a, b and c we have given these factors a minus sign, in deviation from the article quoted in footnote [1]). That q and x must have different signs follows from the fact that the automatic controller has to bring about a change of q counteracting a preceding variation of x.

cording to their form (as function of the time) and according to the point at which they occur in the circuit.

In respect to their form there are continuous and step disturbances. Since the former may be regarded as being the limit of an infinitely large number of infinitesimal steps occurring in succession (cf. fig. 6), it is only necessary to investigate the response of the system to step disturbances. These can therefore be obtained from the square-wave generator employed for adjusting the process analogue.

According to the point at which they occur, one may distinguish between:

1. disturbances in the regulating unit, thus at the input of the process,
2. disturbances somewhere in the course of the process, and
3. disturbances in the detecting element, thus at the output of the process.

There may also be counted among these "disturbances" the changes resulting from the automatic controller being adjusted to a different setting. It is with these distinctions in mind that the square-wave generator is also fitted with a lead for connecting it either to the input of the process (i.e. to the input of the collecting stage S_1) or to the output of the process, for which purpose a second collecting stage, S_2, is provided. If the disturbances introduced at these two points are sufficiently neutralized by the automatic controller then the same will be the case with the disturbances sub (2) occurring somewhere in the course of the process.

Oscillograms obtained with models of continuously acting automatic controllers

Some oscillograms will illustrate the influence of the various terms in the regulation equation (1). The lower curve in fig. 5 represents the step-function response of the process analogue used for the recording of these oscillograms (10 sections of the *R-C* network according to fig. 3).

First some examples are given of a regulating system with proportional action only, with $a = 4$ (12 dB) [5]. Fig. *13-I* shows that the process

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[5] In the model x and q are both voltages and therefore $a = -q/x$ is a non-dimensional quantity. But also in the actual process a is non-dimensional if x is expressed as a percentage of the desired value of the quantity to be regulated and q as a percentage of the corresponding value of the regulating quantity.

The practical advantage of expressing a in decibels is that the total value of a is found by adding up the readings of the various attenuators calibrated in dB (Z_0 and Z_a in fig. 12) and the gain (of A in fig. 12) expressed in dB, which is easier than multiplying.

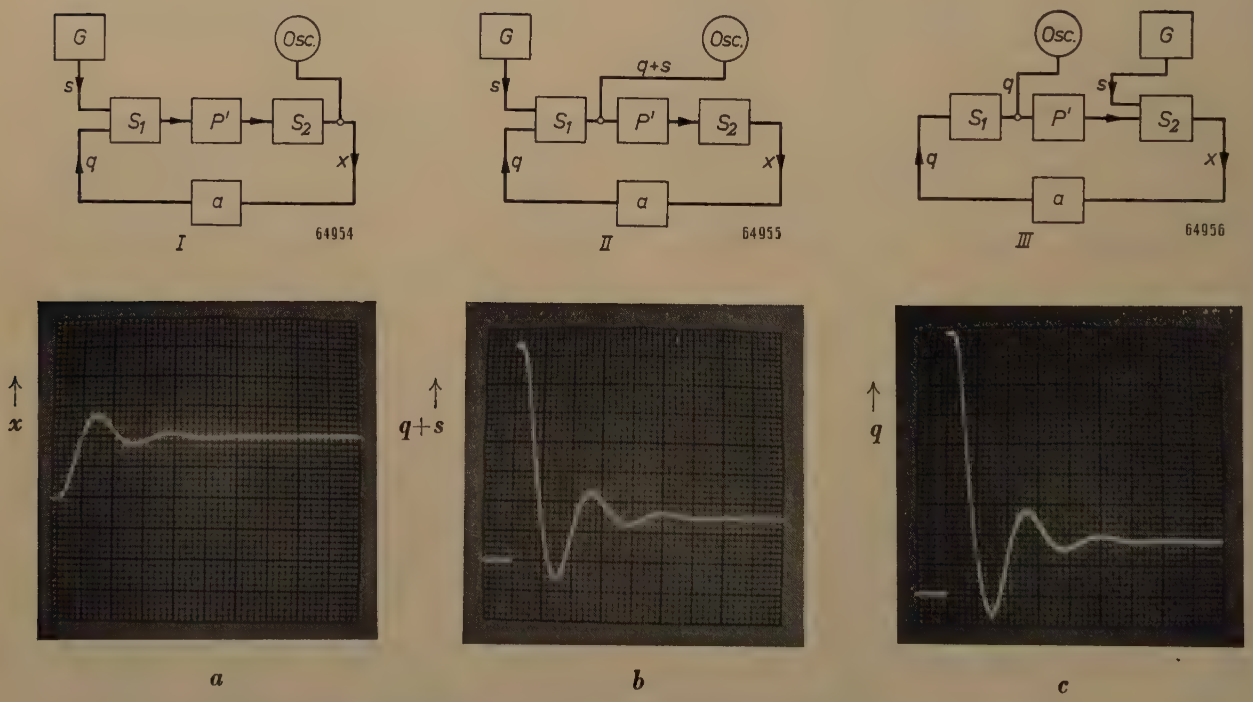

Fig. 13. Oscillograms obtained with the electro-analogue. The step-function response of the process analogue (10 not "decoupled" *R-C* sections) was as represented by the lower curve in fig. 5. The controller had only a proportional element ($a = 4$), as denoted by the rectangle a in the block diagrams *I*, *II* and *III*. In these diagrams it is also indicated at what points the generator G and the oscilloscope *Osc* were connected while the oscillograms underneath were being recorded.

analogue received, via the collecting stage S_1, the sum of the output voltage of the controller $q = -a\,x$ and the step-function voltage s from the square-wave generator, and that the output x from the process analogue was fed into the oscilloscope.

Fig. 13*a* is the oscillogram obtained, on the same scale as fig. 5.

Fig. 13-*II* shows how $q + s$ is fed into the oscilloscope. In the corresponding oscillogram (fig. 13*b*) it is easy to distinguish the step-function voltage s from the square-wave generator, followed by the oscillation of q.

From the oscillograms it is seen that in course of time x and $q + s$ (thus q) become constant. The final values are related to the quantity a in the following way.

The gain of the process analogue (temporarily disconnected from the automatic controller) is so adjusted that a step disturbance with the amplitude s, introduced at the input, causes a change in the output voltage the final of value which, x_∞, is just equal to s. If, in the closed circuit, at the input of the process analogue we have in addition the output voltage q from the automatic controller

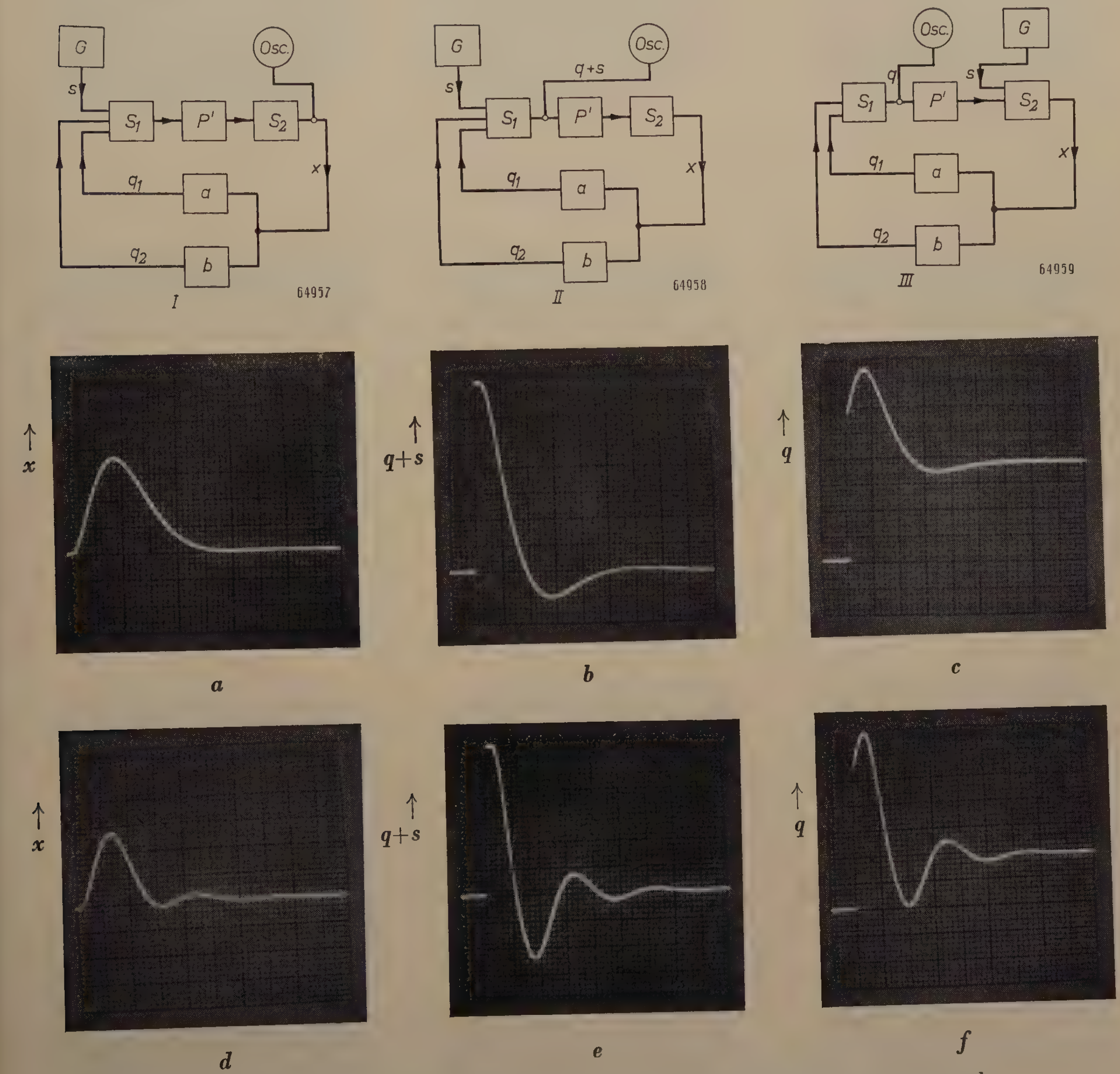

Fig. 14. As in fig. 13 but with an automatic controller having proportional and integral action. For the oscillograms (*a*), (*b*) and (*c*) the values of the coefficients were $a = 1.6$ and $b = 1.25$ millisec^{-1}, whilst for the oscillograms (*d*), (*e*) and (*f*) these were $a = 3.2$ and $b = 3.2$ millisec^{-1}.

with the final value q_∞, then in the stationary state we find:

$$x_\infty = q_\infty + s \quad \ldots \ldots \ldots \quad (2)$$

In the automatic controller the equation

$$q_\infty = -a\,x_\infty \quad \ldots \ldots \ldots \quad (3)$$

holds. From these two equations it follows that

$$x_\infty = \frac{1}{1+a}\,s \quad \ldots \ldots \ldots \quad (4)$$

and
$$q_\infty = \frac{-a}{1+a}\,s \quad \ldots \ldots \ldots \quad (5)$$

From (4) it is seen that the larger the quantity a the smaller x_∞ becomes — i.e. the better a disturbance is neutralized — but that x_∞ never becomes exactly zero (offset). Equation (5) can be numerically verified in fig. 13*b*, for which $a = 4$.

According to fig. 13-*III*, in the collecting stage S_2 the step voltage s is added to the output voltage of the process analogue, the sum being termed x. Since the automatic controller contains only a proportional term the oscillogram of q (fig. 13*c*) is of the same shape as that in fig. 13*b*.

When the automatic controller contains an integrating element (*fig. 14*) then, as already explained in the article quoted in footnote ¹), x is indeed reduced exactly to zero after a disturbance has occurred. With the exception of the integrating element the networks *I*, *II* and *III* of fig. 14 correspond to those of fig. 13. The respective oscillograms in figs 14*a*, 14*b* and 14*c* relate to the case where a = 1.6 (4 dB) and b = 1.25 millisec^{-1}, while the oscillograms in figs 14*d*, 14*e* and 14*f* relate to the case where a = 3.2 (10 dB) and b = 3.2 millisec^{-1}.

The oscillograms in figs 14*c* and 14*f* can be interpreted as follows: owing to the inertia of the process analogue the prescence of this part is not yet noticeable immediately after the step. The oscilloscope thus shows a step $-as$ (in the oscillogram a step upward). This is first followed by a linear rise, corresponding to the presence of the integrating element in the automatic controller, yielding a voltage $-b\int s\cdot dt = -b\,s\,t$ which increases linearly with time. Then, however, comes evidence of the presence

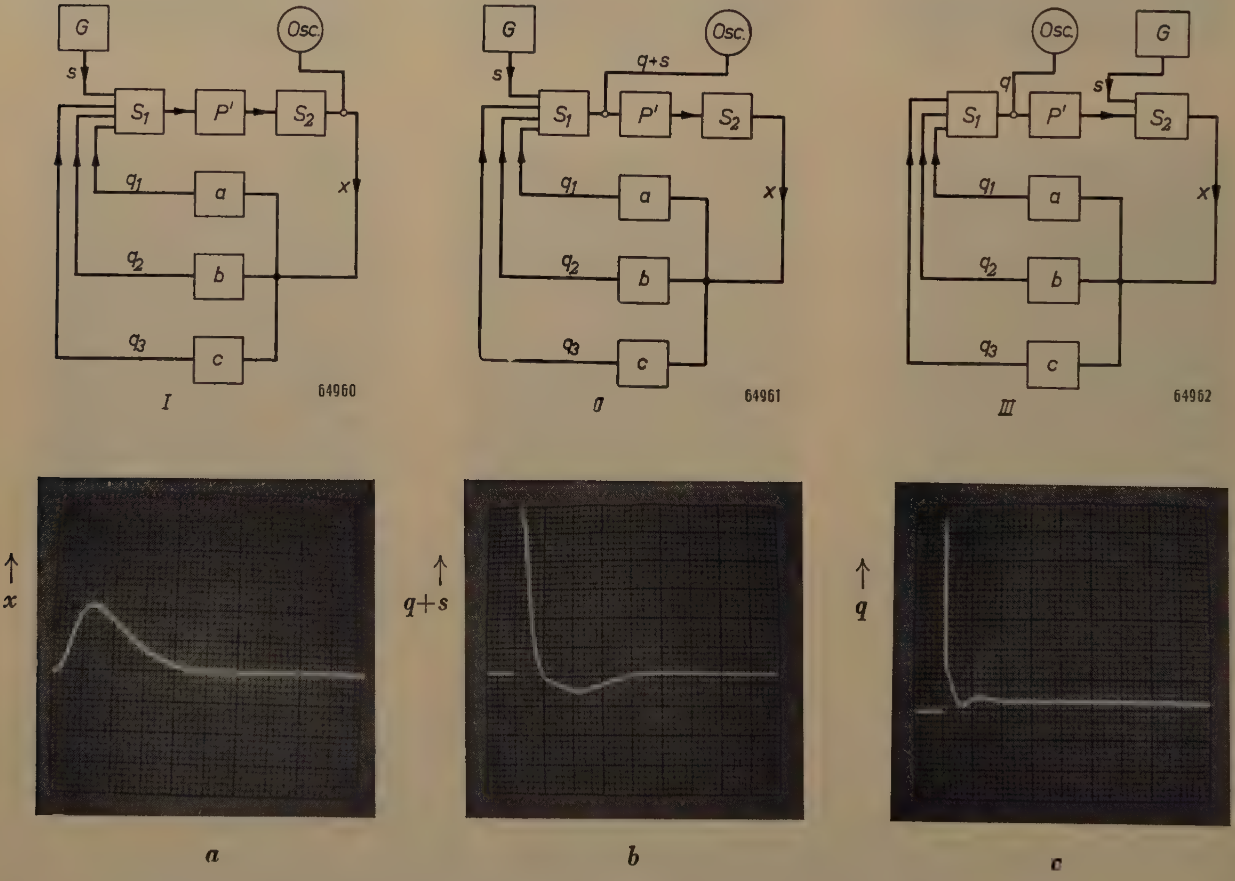

Fig. 15. As in fig. 14 but with also a derivative action in the regulator (a = 3.2, b = 3.2 millisec^{-1}, c = 1 millisec).

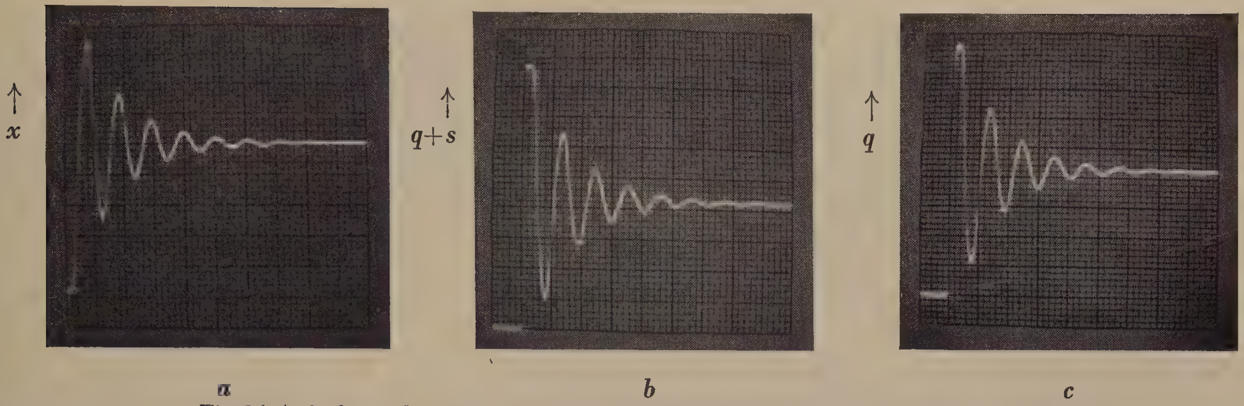

Fig. 16. As in fig. 13, but with the process analogue consisting of a network (10 “decoupled” *R-C* sections, with a step-function response as represented by the upper curve of fig. 5. Here the coefficient a of the proportional term was 1.

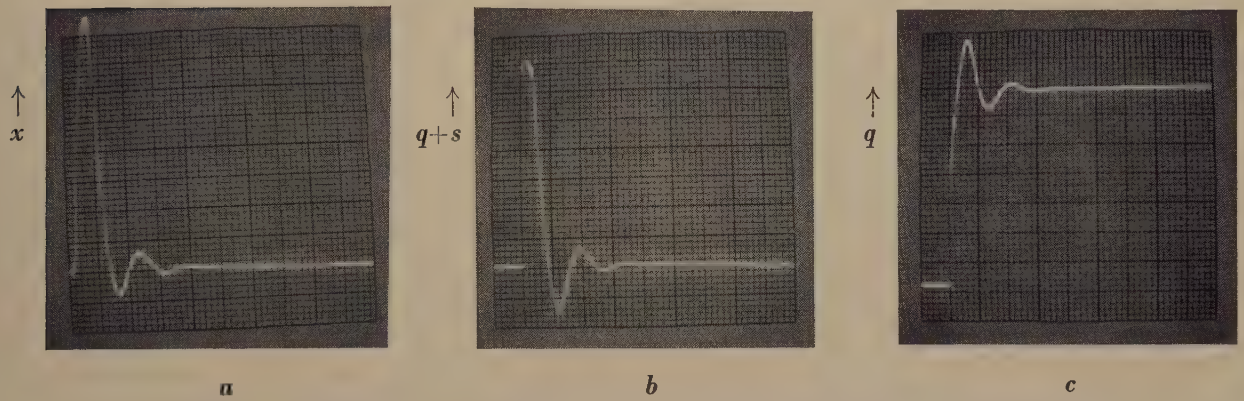

Fig. 17. As in fig. 14 but with a process analogue formed by a network with a step-function response as represented by the upper curve of fig. 5. $a = 0.5$,$b = 2$ millisec^{-1}.

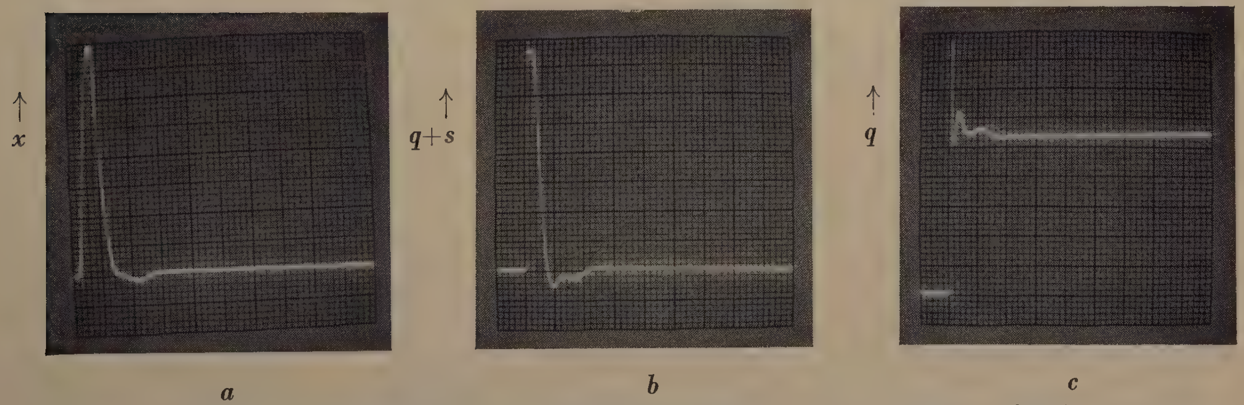

Fig. 18. As in fig. 15 but with a process analogue formed by a network with a step-function response as represented by the upper curve of fig. 5 and with coefficients $a = 0.5$, $b = 2.5$ millisec^{-1}, $c = 0.08$ millisec.

of the process analogue, which now also contributes a certain voltage to the input of the automatic controller. In the final state again equation (2) holds, but since now $x_\infty = 0$ we find $q_\infty = -s$, which means to say that in order to neutralize an interference at the output of the process a permanent change in the position of the regulating unit is necessary. Further the initial step q_0 made by q is equal to $-as$. The ratio of this step to the final value of q, thus q_0/q_∞, is therefore equal to a, as may be seen from figs 14*c* anf 14*f*.

Finally in *fig. 15* similar oscillograms are shown for the case where the automatic controller contains also a differentiating element ($a = 3.2$, $b = 3.2$ millisec^{-1}, $c = 1$ millisec). Apparently the differentiation has a stabilizing effect upon the regulation.

Similar curves, but recorded with a process analogue with a step-function response as shown by the upper curve in fig. 5, are represented in *figs 16, 17* and *18* on the same scale as figs 13, 14 and 15. A comparison of these two groups of oscillograms shows that a process which can be represented by *R-C* sections loading the preceding sections can as a rule be better regulated than a (hypothetical) process corresponding to "decoupled" *R-C* sections; in the former case the regulated quantity gives smaller amplitudes (cf., for example, figs 13*a* and 16*a*), and the position in fig. 16, with a equal to only 1, is decidedly less stable than that in fig. 13 with $a = 4$.

Criteria for the best regulating system

Stability

An idea of the stability can be obtained at once from the oscillograms (e.g. figs 13 and 16): the less the damping of any oscillations present, the less is the stability. The curves do not show, however, whether, in a stable case, any small change in the working conditions or variation of the coefficients a, b and c may already result in instability. (Mechanical wear of parts of the regulating unit, for instance, may well cause a variation of the coefficients.)

A better insight into the degree of stability can be obtained by measuring the stability area. A process with only proportional action ($q_1 = -a\,x$) may become unstable when a exceeds a certain value. If the automatic controller has only an integrating component ($q_2 = -b\int x\,\mathrm{d}t$) then stability may likewise be lost when b becomes too high. An automatic controller having both these functions has, in the case of a certain process, for each value of a a certain limit value of b above which instability occurs; the curve representing this limit value as a function of a forms, with the a-axis, the boundary of the stability area. If the automatic controller has also a differentiating action ($q_3 = -c\,\mathrm{d}x/\mathrm{d}t$) then for every value of the parameter c there is a different relation between the limit values of b and a. For the process analogue whose step-function response is represented by the lower curve in fig. 5, a number of curves determined with the electro-analogue with $b = \mathrm{f}(a)$ and c = constant are given in *fig. 19*.

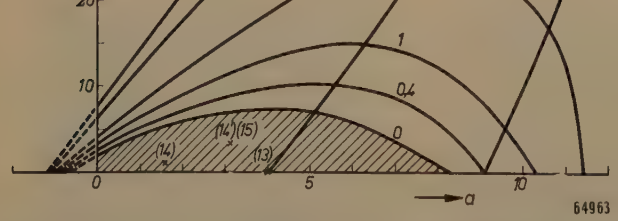

Fig. 19. Values of the coefficient b of the integrating term at which instability just arises, as function of the coefficient a of the proportional term, for constant values of the coefficient c of the differentiating term. This diagram applies for a process having a step function response as represented by the lower curve of fig. 5. The stability zone for $c = 0$ is hatched. The points (*13*), (*14*), (*14'*) and (*15*) correspond to the conditions under which the oscillograms of figures 13, 14*a*, *b*, *c*, 14*d*, *e*, *f* and 15 respectively were recorded.

If practical data are available concerning the variations likely to occur in the coefficients a, b and c, then with the aid of a number of graphs like those in fig. 19 it is possible to investigate whether there is any risk of the system becoming unstable.

Other criteria

Some valuable information is to be gathered from the oscillograms direct, as for instance the magnitude of the first maximum of x, the time that has to elapse after a disturbance before x remains within certain limits, the length of a cycle (if x is oscillatory, etc.

Other data are provided by the performance meter built into the electro-analogue, with which one can determine, as desired, the mean absolute value of x:

$$|\bar{x}| = \frac{1}{T}\int_0^T |x|\cdot \mathrm{d}t\,,$$

or the r.m.s value of x:

$$x_{\mathrm{rms}} = \sqrt{\frac{1}{T}\int_0^T x^2\,\mathrm{d}t}\,.$$

where T is the cycle of the repetition frequency, for which, as already stated, the mains frequency is chosen.

It may be required of an automatic controller that one of these quantities shall be as small as possible. It depends upon the case under investigation whether they are useful as criteria and, if so, which of them forms the best criterion. That is why both possibilities have been provided for in our electro-analogue.

Recording frequency response curves

In the modern theory of regulated systems use is often made of frequency response curves. The behaviour of the system as a whole and also that of its parts is studied with sinusoidal disturbances. Conclusions can be drawn therefrom as to the stability and the quality of the regulation. Literature on this subject has been published in abundance in recent years.

For sinusoidal disturbances to be introduced in the model of the regulating system the electro-analogue has been provided with an *R-C* oscillator the frequency of which is variable. This oscillator can be connected, for instance, to the collecting stage S_1 of the closed regulating circuit (*fig. 20*). In this way sinusoidal voltages arise at all points of the circuit. By measuring the input and output voltages of any element in the circuit one can determine the frequency response of that element. The same can be done for two or more elements connected in cascade, and even for the cascade connection of all the parts making up the circuit. In the latter case it is possible to derive from the measurements the Nyquist diagram, from which, as is known, conclusions can be drawn regarding the stability of the system [6]. Usually the measurements for the Nyquist diagram are taken after the circuit has been opened at some point. If, however, there is an integrator in the automatic controller (or the model) this is not possible, because when the circuit is cut open it becomes unstable: in fact the integral of an accidental step disturbance would continue to increase unrestricted, because in the opened circuit the disturbance is not neutralized. In that case the Nyquist criterion in its original form does not hold. In literature other forms of a wider scope are indicated [7], but it would lead us too far afield to go into these here.

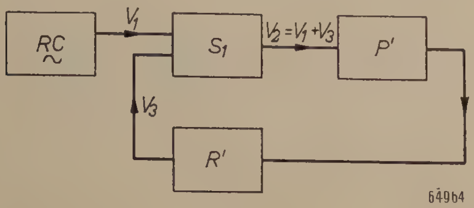

Fig. 20. Arrangement for taking Nyquist diagrams of the closed circuit of the process analogue *P′* and the model *R′* of the automatic controller. The (variable frequency) generator *RC* supplies a sinusoidal voltage V_1, which in the collecting stage S_1 is added to the output voltage V_3 from *R′*.

Models of discontinuously acting automatic controllers

Automatic controller with two-step action

The simplest form of a discontinuously acting automatic controller is that in which the regulating unit can have only two positions (as is the case, for instance, with a thermostat), so that the regulating quantity can assume only two values, Q_1 or Q_2. *Fig. 21a* represents the case — already discussed in the article quoted in footnote [1] — where $Q = Q_1$ so long as the quantity X to be regulated is greater than a critical value X_{cr}, and $Q = Q_2$ so long as $X < X_{cr}$; usually it is so arranged that X_{cr} is just the desired value X_0. Between Q_1 and Q_2 is a value Q_0 at which — if Q could assume that value — under normal working conditions X would have just the desired value X_0. Instead of that, the regulating unit is alternately in the positions Q_1 and Q_2. If it is desired to change over to a different value of X_0 then Q_0 has to be given a different value too. If, however, Q_0 becomes greater than Q_2 or less than Q_1 then the automatic controller no longer functions.

In practice the transition from Q_1 to Q_2 will not usually take place at exactly the same value of X as the change-over from Q_2 to Q_1: owing to backlash in the mechanical parts of the regulating unit for instance, or owing to the difference between the current at which a relay is closed and that at which it is opened, the transitions take place at different values of X (X_1 and X_2) either side of X_0. Instead of fig. 21*a* we then get fig. 21*b*. Denoting the deviations of Q and X from the equilibrium values again by q and x, then $q = Q - Q_0$ and $x = X - X_0$; the regulating cycle $q = f(x)$ for an automatic controller with two-step action having backlash then has the shape shown in fig. 21*c*.

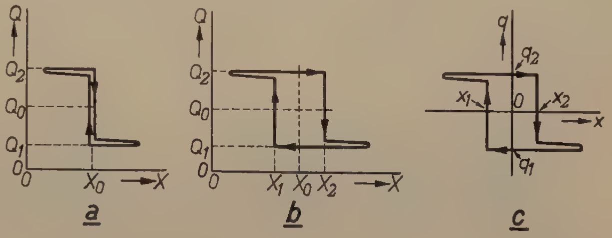

Fig. 21. Automatic controllers with two-step action, (*a*) without, (*b*) and (*c*) with backlash. In case (*a*) the regulating unit takes up the position corresponding to $Q = Q_1 < Q_0$ when X is greater than the desired value X_0, and $Q = Q_2 > Q_0$ when $X < X_0$ (Q_0 is the — impossible — position in which under normal working conditions X would assume the value X_0). *b*): The same but with a backlash X_1-X_2. In (*c*) the quantities $q = Q - Q_0$ and $x = X - X_0$ have been plotted against each other for the case (*b*).

The presence of backlash naturally reduces the response sensitivity and the accuracy of the regulation, but there is also a good side to it. In the case of an automatic controller without any backlash the regulating unit takes up the other position as soon as there is the least difference between X and X_{cr}, whereas in the case of a controller with backlash this does not take place until that difference reaches a certain value. In the latter case, therefore, the regulating unit is less subject to mechanical wear, and that is why sometimes some backlash is purposely given to the regulating unit.

It appears to be possible to build up electric circuits which show a relation between the output and the input voltages similar to that existing between q and x according to the hysteresis loop of fig. 21*c*. Such a circuit can serve as an electrical model for an automatic controller with two-step action. If, for instance, a sinusoidal voltage (x in

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[6] H. Nyquist, Regeneration theory, Bell Syst. techn. J. **11**, 126-147, 1932. See also B. D. H. Tellegen, Philips Techn. Rev. **2**, 292, 1937.

[7] H. Bode, Network analysis and feedback amplifier design (Chapter VIII), New York 1945, and F. Strecker, Die elektrische Selbsterregung, Stuttgart 1947.

fig. 22) is applied to this circuit then a square-wave output voltage (q) is obtained.

In the electro-analogue containing such a circuit the latter can be combined with the process analogue to form a closed regulating circuit (*fig. 23*), in which also an amplifier, variable attenuators and a collecting stage are included. The limit $x_2 = -x_1$ is governed in the circuit by a direct voltage; thus by changing this voltage it is possible to vary the width of the loop. Via the collecting stage a direct voltage s can be added, corresponding to a variation of Q_0 or to bringing about a variation of Q_0.

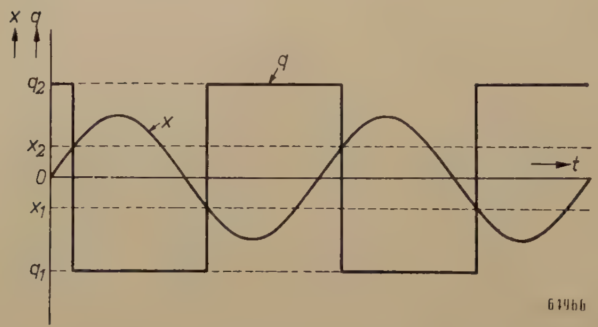


Fig. 22. When a sinusoidal voltage x (with amplitude greater than the limits $x_2 = -x_1$ of the backlash) is applied to an electric network forming the model of an automatic controller with two-step action there arises at the output of the network a square-wave voltage q.

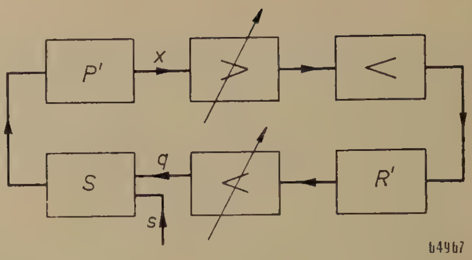


Fig. 23. Regulating circuit for studying discontinuously acting controlers (model R'). P' process analogue. Via the collecting stage S a direct voltage s can be applied to form the analogue of a disturbance or of a transition to some other desired value in the actual controller.

In *figs 24a, b* and *c* oscillograms of q and x are represented respectively for $s = 0$, $s = +5$ V and $s = -5$ V, where changes takes place in the ratio of the intervals in which q has the value q_1 or q_2 respectively.

Automatic controller with three-step action

By giving the regulating unit a third position, Q_0, in between the positions Q_1 and Q_2 (*fig. 25*) one has the advantage that the system is at rest so long as the value of X is between X_1 and X_2.

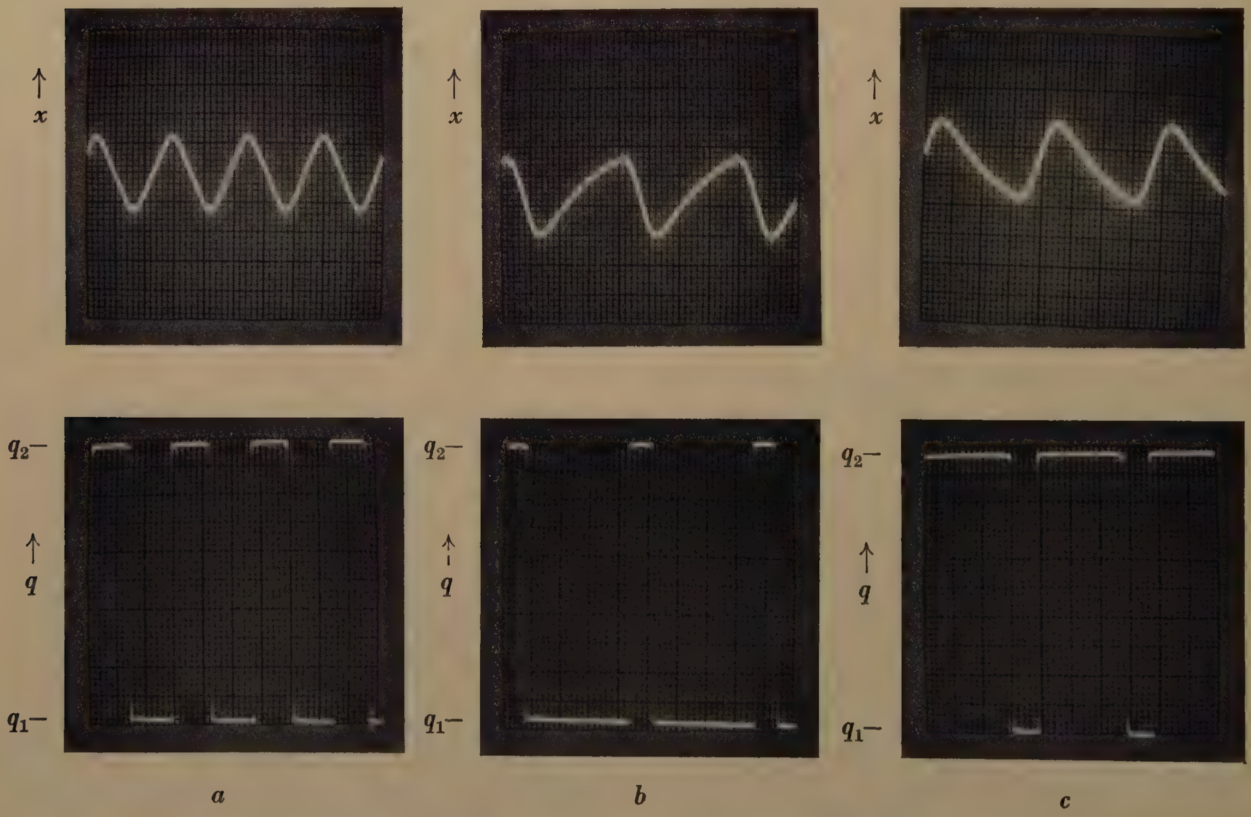


Fig. 24. Oscillograms obtained with a circuit according to fig. 23 with an automatic controller with two-step action. In the case (a) $s = 0$ and thus Q_0 lies halfway between Q_1 and Q_2 (fig. 21b). The intervals during which $q = q_1$ and q_2 respectively are of equal length. In the case (b) s is positive and in case (c) equally negative; the intervals during which $q = q_1$ and q_2 respectively are here unequal.

so that it suffers less from wear than a system lacking a state of rest. If considerable deviations from the normal working conditions arise only at long intervals then the controller and the regulating unit can remain at rest for a long time.

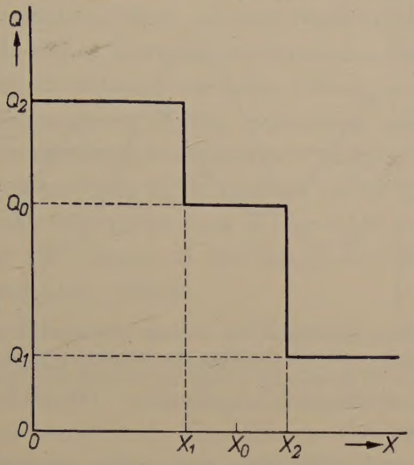
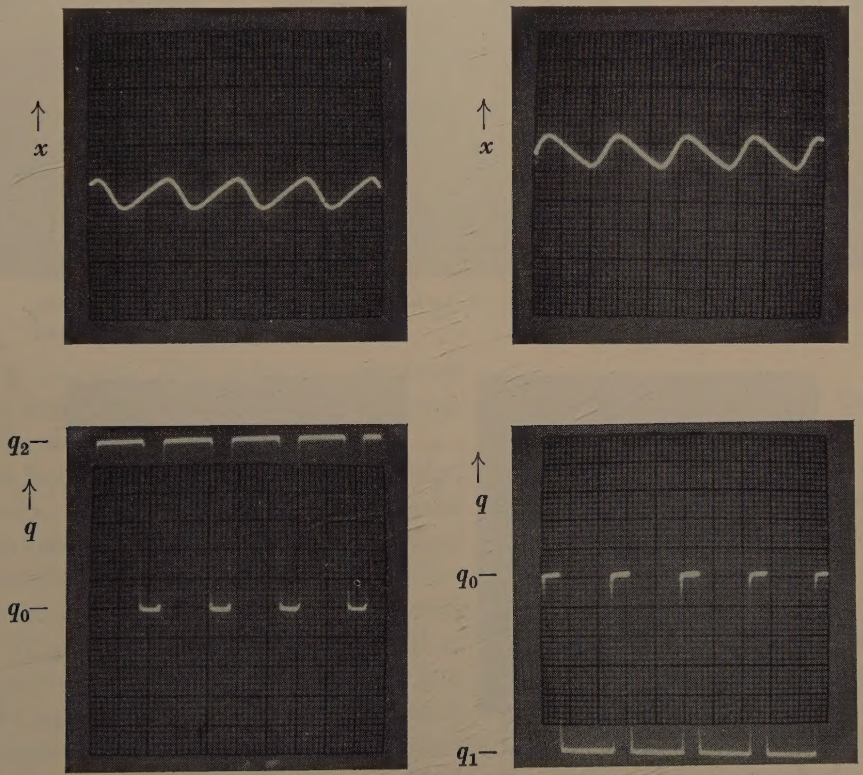


Fig. 25. Q as a function of X for an automatic controller with three step-action: in addition to the positions Q_1 and Q_2 also the intermediate position Q_0 is now possible. The regulating unit will be in the position Q_0 so long as X_0 remains between X_1 and X_2.

If, for some reason or other, X exceeds the value X_2 then the controlling unit causes the regulating unit to move to and fro between the position of rest Q_0 and the lower position Q_1, thereby correcting the too high value of X. The functioning can then be regarded as being similar to that of an automatic controller with two-step action (positions Q_0 and Q_1). If, on the other hand, X drops below X_1 then the controlling unit will fluctuate between the position Q_0 and the higher position Q_2, so that apparently we again have to do with an automatic controller with two-step action, but this time with the positions Q_0 and Q_1. Since the differences $Q_2 — Q_0$ and $Q_0 — Q_1$ are smaller than the difference $Q_2 — Q_1$ in the case of a controller with two-step action, also the periodic fluctuation of X caused by the changes of the controlling unit is smaller. This can also be regarded as an important advantage of an automatic controller with three-step action over one with two-step action. The fluctuation referred to can be reduced still further by increasing the number of positions of the controlling unit. The higher the number of positions the more closely the curve of echelon shape representing the relation between Q and X approaches a straight line, and the more the working of the automatic controller approaches that of a continuous controller with proportional action.

The functioning of an automatic controller with



a *b*

Fig. 26. Oscillograms obtained with a circuit according to fig. 23 with an automatic controller with three-step action, (*a*) with positive, (*b*) with equally negative disturbance voltage *s*.

three-step action is illustrated by the oscillograms in *fig. 26*. In this case, by applying a positive or a negative voltage s to the input of the process analogue, the rest level Q_0 was shifted in such a way that the controller came into action and moved to and fro between the positions Q_0 and Q_2, respectively between the positions Q_0 and Q_1.

Multi-speed floating control

Instead of the regulating unit being so made that it can only take up certain positions, it can be arranged so as to run with a certain, constant, speed dq/dt in the one direction or in the other or remain at rest according to the value of x. For instance:

$$dq/dt = -p \text{ when } x > x_2,$$

$$dq/dt = 0 \text{ when } x_2 > x > x_1,$$

and $$dq/dt = +p \text{ when } x < z_1.$$

This is made technically possible, for instance, by having a controlling unit with three-step action driving a servomotor which operates the regulating unit and turns at a constant speed in one direction or the other or remains at rest according to the position of the controlling unit. The motor then adjusts the regulating unit by an amount q varying linearly with the time: $q = \int p \, dt = pt + \text{const.}$

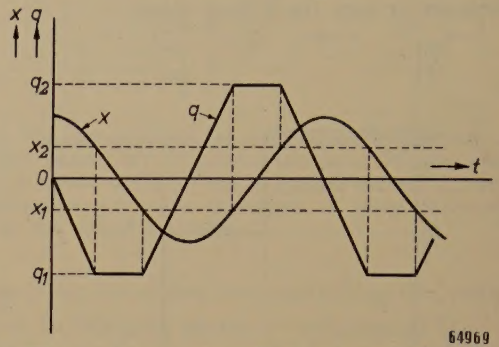
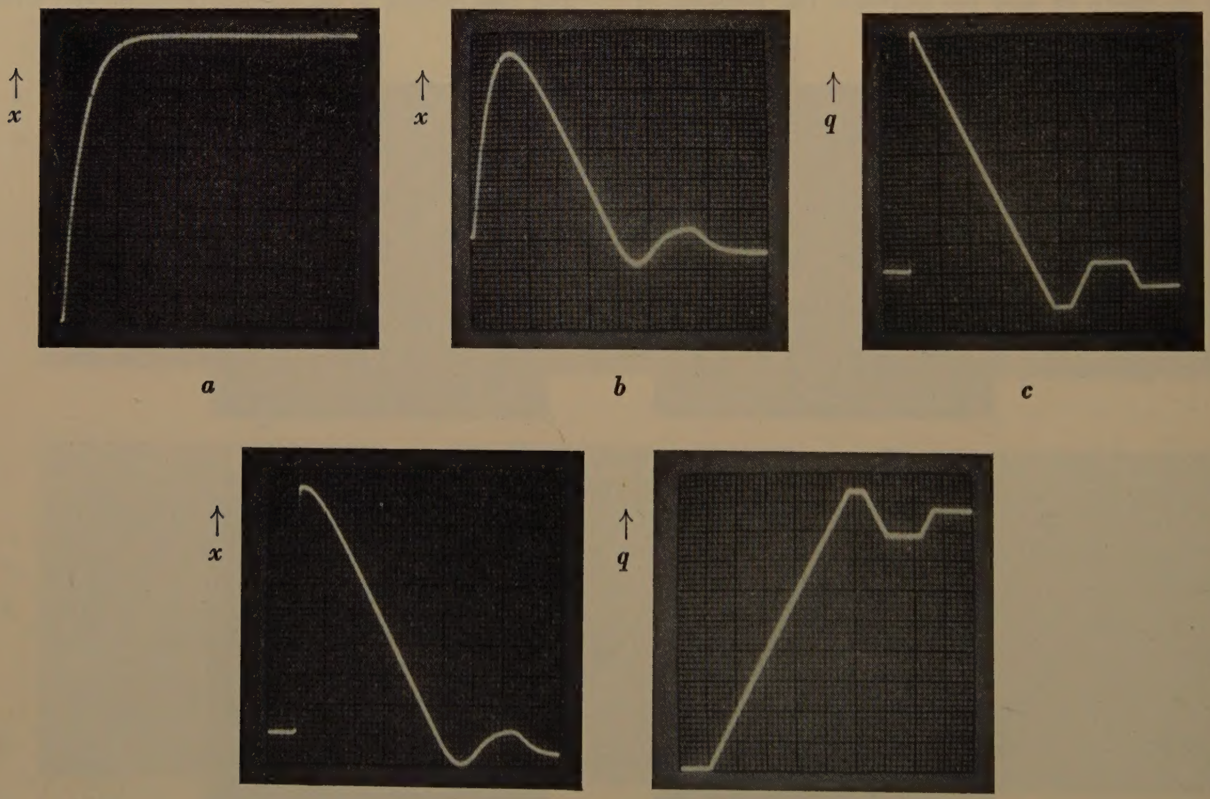


Fig. 27. When an alternating voltage x is applied to the input of an electric network forming the analogue of an automatic controller with three-step action the output voltage q assumes the shape of the drawn trapezium.

Compared with the automatic controller with three-step action, the multi-speed floating control has the advantage that in the event of permanent change from the normal (average) working conditions the motor provides for a permanent readjustment of the regulating unit.



a b c

d e

Fig. 28. *a*) Step-function response of a process analogue which together with an automatic controller with three-step action formed a regulating circuit from which the oscillograms (*b*) - (*e*) have been recorded. (*b*) and (*c*) represent the variation of x and q respectively in the case of a step disturbance at the input of the process, (*d*) and (*e*) the same quantities when a disturbance is applied at the output.

In an electrical model the function of the motor can be performed by an integrating network. When the output voltage (—p, $+p$ or 0) from a model of an automatic controller with three-step action is applied to an integrating network then at the output of that network there arises a voltage q which either increases or decreases linearly with the time or remains constant, thus in analogy with the readjustment of the regulating unit.

If, for instance, a sinusoidal voltage (with amplitude greater than x_2) is applied to the input of the automatic controller then it is easy to see (*fig. 27*) that at the output of the integrator there will be a trapezoidal voltage.

Fig. 28 shows some oscillograms recorded with a regulating circuit consisting of a process analogue, an automatic controller with three-step action and an integrator. Fig. 28*a* is the step-function response of the process analogue employed. A step disturbance at the input of the process analogue (corresponding to a sudden readjustment of the regulating unit) produces the variation x in the regulated quantity as illustrated in fig. 28*b*, and this in turn causes the voltage q at the input of the process analogue to change according to fig. 28*c*. The consequences of a step disturbance at the output of the process analogue are shown in figs 28*d* and *e*, respectively at the input and at the output of the process. In both these cases in the final position x comes to rest at a value between x_1 and x_2.

The examples that have been given here should be sufficient to show how valuable an aid the electro-analogue is for studying regulating processes. Details of the electric circuits of various parts such as the universal process analogue, the collecting stages, integrators and differentiators have purposely been omitted, since they will form the subject of a second article.

Summary. An electro-analogue is an apparatus with which electrical models of regulating devices can be built up and studied. The electro-analogue discussed here comprises two special models and one universal model of the process to be regulated, the latter model consisting of an electric network — mainly a delay network — the step-function response of which, displayed on the screen of an oscilloscope, can be given a shape identical to that of any process to be imitated. Further the electro-analogue comprises electrical models of automatic controllers with continuous action and of certain controllers with discontinuous action. In the model of the continuously acting controller a proportional term, a single and a double integrating term and a single and a double differentiating term can be realized.

After the model of the process has been given the right characteristic it is combined with one of the models of the automatic controller to form a closed regulating circuit. By applying a step disturbance at a suitable point it is possible to study with the aid of oscillograms the behaviour of the circuit at different values of the parameters. In particular it can be investigated under what conditions instability arises and how high the quality of the regulation is. Oscillograms are given for various cases, both with continuously and with discontinuously acting controllers.

A description of the electric circuits of the principal parts will be given in a subsequent article.

ABSTRACTS OF RECENT SCIENTIFIC PUBLICATIONS OF THE N.V. PHILIPS' GLOEILAMPENFABRIEKEN

Reprints of these papers not marked with an asterisk can be obtained free of charge upon application to the Administration of the Research Laboratory, Kastanjelaan, Eindhoven, Netherlands.

1934: W. de Groot: The radiation constants and the light equivalent of energy (Physica, The Hague **16**, 419-420, 1950, No. 4).

Hoffmann has drawn attention to the fact that the discrepancy between the experimental and the theoretical values of the Stephan-Boltzmann constant σ and the Planck constant c_2 would disappear simultaneously if the figure for the freezing point of gold (T_{Au}) were a few degrees higher than the internationally adopted value of 1336 °K. It is shown that the same holds true for the discrepancy between the experimental and the theoretical values of the light-equivalent of energy. At the same time it is shown why Van Dusen and Dahl obtained an experimental c_2-value in accordance with the theoretical value.

1935: C. J. Bouwkamp: On integrals occurring in the theory of diffractions of electromagnetic waves by a circular disk (Proc. Kon. Ned. Akad. Wetensch. Amsterdam **53**, 654-661, 1950, No. 5, Indigationes Mathematicae **12**, 199-206, 1950, No. 3).

Evaluation of integrals occurring in the rigorous theory of diffraction of a plane-polarized electromagnetic wave by a circular disk or by a circular hole in an infinite plane screen, if the radius of the disk or hole is small compared to the wavelength. Expansions are given in terms of Legendre functions, Bessel functions, and hypergeometric functions.

1936: C. J. Bouwkamp: On the theory of spheroidal wave functions of order zero (Proc. Kon. Ned. Akad. Wetensch. Amsterdam **53**, 931-944, 1950, No. 6; Indigationes Mathematicae **12**, 326-339, 1950, No. 3).

Spheroidal wave functions of order zero emerge from a separation of the wave equation in spheroidal coordinates in the case of axially symmetric wave problems. The author gives an account of the conventional theory of these functions. In addition, he draws attention to a new canonical system of spheroidal wave functions which has many advantages over the conventional systems.

1937: W. J. Oosterkamp: Dose measurements on contact therapy tubes (Acta Radiologica **33**, 491-506, 1950, No. 6).

The various causes of errors in contact-therapy dosimetry are analysed and methods for avoiding these errors are described. Extensive measurements have been carried out with the Philips apparatus in regard to: the absorption characteristic of the radiation in aluminium, the half-value layer for different added filters, the field distribution where the effect of a compensating filter is described, the determination of depth-dose data and of isodose curves in a water phantom, the influence of secondary radiation from the applicators and the dose rate of the stray-radiation around the tube caused by scattering from the patient. The dynamic electrometer used proved to be very useful for this kind of work. The results of the measurements are compared with those obtained by other authors.

1938: H. O. Huisman: Investigations into the vitamin A series, I. A new synthesis of vitamin A acid and vitamin A (Rec. Trav. chim. Pays-Bas **69**, 851-857, 1950, No. 6).

The chemical and physical properties of β-ionylidene acetaldehyde, the important key intermediate for the synthesis of vitamin A, are given. Starting from β-ionylidene acetaldehyde a new synthesis of vitamin A acid and vitamin A is described.

1939*: W. Elenbaas, J. Funke, Th. Hehenkamp, L. C. Kalff, A. A. Kruithof, J. L. Ouweltjes, L. M. C. Touw, D. Vermeulen, R. van der Veen: Fluorescentie-verlichting (edited by Prof. Dr. C. Zwikker), Meulenhoff, Amsterdam 1950 (262 pp., 173 fig.). (Fluorescent lighting, English edition in preparation.)

See Philips Techn. Rev. **12**, 369, 1950, No. 12.

1940: B. D. H. Tellegen: De krachten tussen twee stroomgeleiders (T. Ned. Radiog. **15**, 157-161, 1950, Nos 4 and 5). (The forces between two steady currents; in Dutch.)

The forces between two steady currents can be conceived of as the resultant of fictitious attracting forces and fictitious couples which the elements of current exert on one another; the attracting forces are proportional to the scalar product of the current elements divided by the square of their distance; the couples are proportional to the vector product of the current elements divided by their distance.

1941: J. H. van Santen and G. H. Jonker: Electrical conductivity of ferromagnetic compounds of manganese with perovskite structure (Physica, The Hague **16**, 599-600, 1950, Nos 7 and 8).

Report on measurements of electrical conductivity of polycrystalline ceramic compounds previously studied (see No. **1932** of these reports). Graphs are given showing log ϱ versus $1/T$ for $(La_{1-x}\ Sr_x)MnO_3$ with x as parameter and log ϱ versus x for $T = 100$ °K. A striking resemblance is to be observed between the curves showing the Curie temperature and saturation magnetisation versus composition. Similar sets of curves have been obtained for systems in which Sr is replaced by Ca or Ba.

1942: F. de Boer, J. H. van Santen and E. J. W. Verwey: The electrostatic contribution to the lattice energy of some ordered spinels (J. chem. Phys. **18**, 1032-1034, 1950, No. 8).

The electrostatic contribution to the lattice energy of some ordered spinels has been calculated. It is made plausible that also in the case of certain spinels that do not show any long-range ordering a considerable amount of short-range ordering is to be expected. As a consequence the lattice energy of "inversed" spinels containing different ions at octahedral positions is increased with respect to the corresponding "normal" spinels containing one type of ions at octahedral interstices.

1943*: E. J. W. Verwey: The role of the electric double layer in the behaviour of lyophobic colloids (Colloid chemistry — theoretical and applied, Vol. 7, Reinhold Publ. Corp., New York 1950; p. 47-66).

Discussion of the properties of the electric double layer of colloid particles and its influence on the interaction of particles (see Nos **1730**, **1769***, **1916** of these abstracts).